5. [12 points] Franklin, your robot, goes to the local store and buys a new chef’s knife. The handle of the knife is given by the region contained between the lines $y = 7$, $y = 4$, $x = 0$ and $x = 10$. The blade of the knife is in the shape of the region bounded by the line $x = 10$, $y = 7$ and the curve $y = \frac{7(x-10)^2}{400}$. Assume all lengths are in centimeters. Below is a diagram of the knife.

Assume that the density of the knife is constant, with value $\delta$ kg/cm$^2$.

a. [2 points] Find the total mass of the handle of the knife. Include units.

Solution:

Mass of handle $= 3 \cdot 10 \cdot \delta = 30\delta$ kilograms

b. [4 points] Write an expression involving integrals that gives the total mass of the blade of the knife. Do not evaluate any integrals.

Solution:

Mass of blade $= \int_{10}^{30} \delta \left(7 - \frac{7(x-10)^2}{400}\right)dx = \frac{280}{3} \delta$ kilograms

c. [2 points] Write an expression involving integrals that gives the $x$-coordinate of the center of mass of the blade portion of the knife. Do not evaluate any integrals.

Solution:

$\bar{x} = \frac{\int_{10}^{30} x \left(7 - \frac{7(x-10)^2}{400}\right)dx}{\int_{10}^{30} \left(7 - \frac{7(x-10)^2}{400}\right)dx}$

d. [4 points] Write an expression involving integrals that gives the $x$-coordinate of the center of mass of the whole knife (the blade and handle together). Do not evaluate any integrals.

Solution:

$\bar{x} = \frac{\int_{0}^{10} 3xdx + \int_{10}^{30} x \left(7 - \frac{7(x-10)^2}{400}\right)dx}{30 + \int_{10}^{30} \left(7 - \frac{7(x-10)^2}{400}\right)dx}$