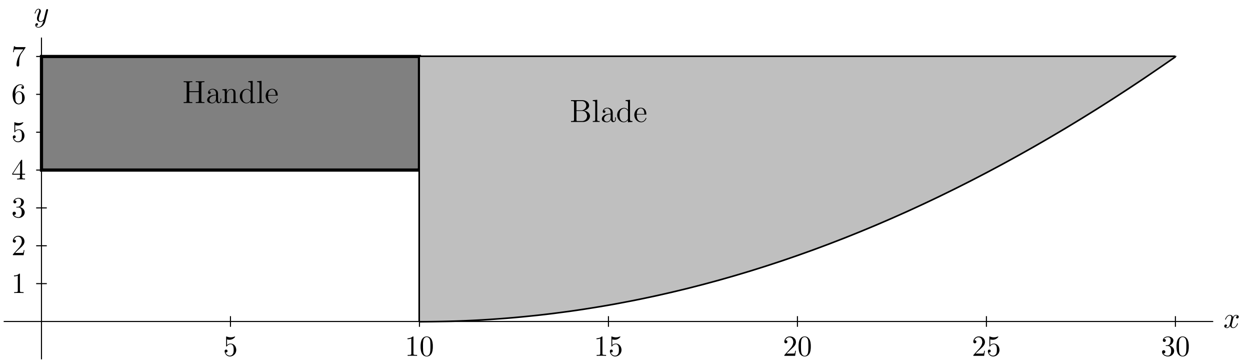
5. [12 points] Franklin, your robot, goes to the local store and buys a new chef's knife. The handle of the knife is given by the region contained between the lines $y = 7$, $y = 4$, $x = 0$ and $x = 10$. The blade of the knife is in the shape of the region bounded by the line $x = 10$, $y = 7$ and the curve $y = \frac{7(x-10)^2}{400}$. Assume all lengths are in centimeters. Below is a diagram of the knife.



Assume that the density of the knife is constant, with value δ kg/cm^2.

a. [2 points] Find the total mass of the **handle of the knife**. Include units.

Solution:

$$\text{Mass of handle} = 3 \cdot 10 \cdot \delta = 30\delta \quad \text{kilograms}$$

b. [4 points] Write an expression involving integrals that gives the total mass of the **blade of the knife**. Do not evaluate any integrals.

Solution:

$$\text{Mass of blade} = \int_{10}^{30} \delta(7 - \frac{7(x-10)^2}{400})dx = \frac{280}{3}\delta \quad \text{kilograms}$$

c. [2 points] Write an expression involving integrals that gives the x-coordinate of the center of mass of the **blade portion of the knife**. Do not evaluate any integrals.

Solution:

$$\bar{x} = \frac{\int_{10}^{30} x(7 - \frac{7(x-10)^2}{400})dx}{\int_{10}^{30} 7 - \frac{7(x-10)^2}{400}dx}$$

d. [4 points] Write an expression involving integrals that gives the x-coordinate of the center of mass of the **whole knife** (the blade and handle together). Do not evaluate any integrals.

Solution:

$$\bar{x} = \frac{\int_{0}^{10} 3x dx + \int_{10}^{30} x(7 - \frac{7(x-10)^2}{400})dx}{30 + \int_{10}^{30} 7 - \frac{7(x-10)^2}{400}dx}$$