

6. [11 points] Franklin, your robot, is zipping around the kitchen making his famous “Definitely Not Poison!” soup. His coordinates in the  $xy$ -plane are given by the parametric equations

$$x = t^2 - t \quad y = -\sin(\pi t)$$

$t$  seconds after he starts making soup. Assume that both  $x$  and  $y$  are measured in meters.

- a. [2 points] Calculate  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ .

$$\frac{dx}{dt} = \underline{2t - 1} \quad \frac{dy}{dt} = \underline{-\pi \cos(\pi t)}$$

- b. [2 points] Find all times  $t$  when Franklin’s velocity is zero.

*Solution:* Franklin comes to a stop at all times  $t$  when both  $\frac{dx}{dt} = 0$  and  $\frac{dy}{dt} = 0$ .

- $\frac{dx}{dt} = 2t - 1 = 0$  when  $t = 1/2$ .
- $\frac{dy}{dt} = -\pi \cos(\pi t) = 0$  when  $t = 1/2, 3/2, 5/2$ , etc.

So Franklin comes to a stop when  $t = 1/2$ .

$$t = \underline{\hspace{2cm} 1/2 \hspace{2cm}}$$

- c. [3 points] Find Franklin’s **speed** when  $t = 2$  seconds. Include units.

*Solution:*

$$\text{Franklin's speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

When  $t = 2$ :

- $\frac{dx}{dt} = 2(2) - 1 = 3$
- $\frac{dy}{dt} = -\pi \cos(2\pi) = -\pi$

Franklin’s speed when  $t = 2$  is  $\sqrt{3^2 + \pi^2} \approx 4.34$  meters per second.

$$\text{Franklin's speed} = \underline{\hspace{2cm} \sqrt{3^2 + \pi^2} \approx 4.34 \text{ m/s} \hspace{2cm}}$$

- d. [4 points] Write an integral which gives the distance traveled by Franklin during his first five seconds of zipping around. Do not evaluate this integral.

*Solution:*

$$\int_0^5 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^5 \sqrt{(2t - 1)^2 + (-\pi \cos(\pi t))^2} dt$$