6. [11 points] Franklin, your robot, is zipping around the kitchen making his famous “Definitely Not Poison!” soup. His coordinates in the $xy$-plane are given by the parametric equations

$$x = t^2 - t \quad y = -\sin(\pi t)$$

t seconds after he starts making soup. Assume that both $x$ and $y$ are measured in meters.

a. [2 points] Calculate $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$$\frac{dx}{dt} = 2t - 1 \quad \frac{dy}{dt} = -\pi \cos(\pi t)$$

b. [2 points] Find all times $t$ when Franklin’s velocity is zero.

Solution: Franklin comes to a stop at all times $t$ when both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

- $\frac{dx}{dt} = 2t - 1 = 0$ when $t = 1/2$.
- $\frac{dy}{dt} = -\pi \cos(\pi t) = 0$ when $t = 1/2, 3/2, 5/2, \text{etc.}$

So Franklin comes to a stop when $t = 1/2$.

$$t = \frac{1}{2}$$

c. [3 points] Find Franklin’s speed when $t = 2$ seconds. Include units.

Solution:

Franklin’s speed = $\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$

When $t = 2$:

- $\frac{dx}{dt} = 2(2) - 1 = 3$
- $\frac{dy}{dt} = -\pi \cos(2\pi) = -\pi$

Franklin’s speed when $t = 2$ is $\sqrt{3^2 + \pi^2} \approx 4.34$ meters per second.

Franklin’s speed=$\sqrt{3^2 + \pi^2} \approx 4.34$ m/s

d. [4 points] Write an integral which gives the distance traveled by Franklin during his first five seconds of zipping around. Do not evaluate this integral.

Solution:

$$\int_0^5 \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_0^5 \sqrt{(2t - 1)^2 + (-\pi \cos(\pi t))^2} dt$$