7. [13 points] Kazilla is designing a new board game. She is interested in using the region R in the xy-plane bounded by $y = 2$, $y = x$, $x = 1$ and $x = 0$.

a. [4 points] The first part of the game is a spinning top formed by rotating the region R around the y-axis. Write an integral (or a sum of integrals) that gives the volume of the spinning top. Do not evaluate your integral(s).

Solution: Shell method:

$$\int_0^1 2\pi(2-x)x\,dx$$

Washer method:

$$\int_0^1 \pi y^2\,dy + \int_1^2 \pi\,dy$$

b. [4 points] Another game piece has a base in the shape R, but with semicircular cross sections **perpendicular** to the x-axis. Write an integral which gives the volume of the game piece. Do not evaluate your integral.

Solution:

$$\frac{\pi}{8}\int_0^1 (2-x)^2\,dx$$

c. [5 points] A third game piece has volume given by $\displaystyle\int_0^2 \pi(h(x))^2\,dx$ where $h(x)$ is a continuous function of x. Use MID(3) to approximate the volume of this third game piece. Be sure to write out all of the terms in your approximation. Your answer may contain the function $h(x)$.

Solution:

$$MID(3) = \frac{2}{3}[\pi(h(1/3))^2 + \pi(h(1))^2 + \pi(h(5/3))^2]$$