1. [16 points] Suppose $g(x)$ is a function with the following properties:

- $\displaystyle\int_5^1 g(x)dx = 7.$
- $\displaystyle\int_3^5 g(x)dx = -3.$
- $g(x)$ is odd.

In addition, a table of values for $g(x)$ is given below.

x	0	1	2	3	4	5
$g(x)$	0	2	-1	-3	-1	1

Calculate (a)-(c) **exactly**. Show your work and do not write any decimal approximations.

a. [4 points] $\displaystyle\int_1^{\sqrt{3}} xg(x^2)dx.$

Solution: Using the substitution $w = x^2, dw = 2xdx$, $\int_1^{\sqrt{3}} xg(x^2)dx = \int_1^3 \frac{1}{2}g(w)dw = \frac{1}{2}\left(\int_1^5 g(w)dw + \int_5^3 g(w)dw\right) = \frac{1}{2}(-7+3) = -2.$

b. [4 points] $\displaystyle\int_1^5 xg'(x)dx.$

Solution: Using integration by parts, $u = x, dv = g'(x)dx$, so $du = dx, v = g(x)$, the integral $\int_1^5 xg'(x)dx = [xg(x)]_1^5 - \int_1^5 g(x)dx = (5g(5) - g(1)) - (-7) = 10.$

c. [3 points] The average value of $g(x)$ on $[-5, -1]$.

Solution: The average value is $\frac{1}{-1-(-5)}\int_{-5}^{-1} g(x)dx$. Since g is odd, this is equal to $\frac{-1}{4}\int_1^5 g(x)dx = \frac{-1}{4}(-7) = \frac{7}{4}.$

d. [5 points] Approximate

$$\int_2^4 xg(x)dx$$

using TRAP(2). Write out all the terms of your sum and your final answer.

Solution:

$$\text{LEFT}(2) = \frac{4-2}{2}(2g(2) + 3g(3)) = -11$$
$$\text{RIGHT}(2) = \frac{4-2}{2}(3g(3) + 4g(4)) = -13$$
$$\text{TRAP}(2) = \frac{1}{2}(\text{LEFT}(2) + \text{RIGHT}(2)) = -12.$$