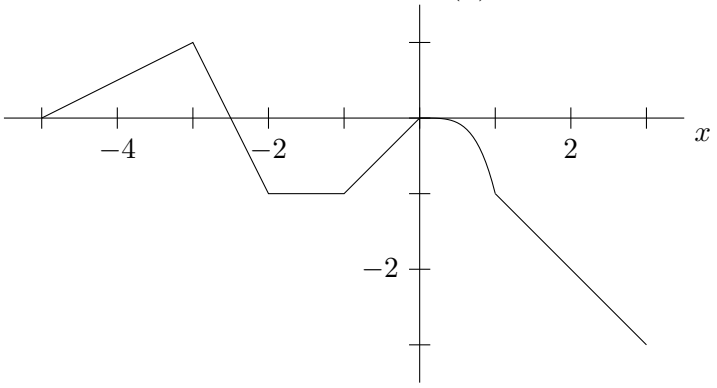
5. [11 points] The following is the graph of a function $g(x)$.



Note that $g(x)$ is piecewise linear on [-5,0] and linear on [1,3].

a. [5 points] Estimate $\int_1^3 e^{-g(x)}dx$ using MID(2). Write out all the terms of your sum as well as your final answer.

Solution:

$$\text{MID}(2) = \frac{3-1}{2}\left(e^{-g(1.5)} + e^{-g(2.5)}\right) = e^{1.5} + e^{2.5} \approx 16.664$$

b. [3 points] If you were estimating $\int_0^1 \frac{dx}{g(x)+2}$ using LEFT(n), would your estimate be an overestimate, underestimate, or is there not enough information to tell? Circle your answer. You do not need to show your work or explain your answer.

Overestimate **Underestimate** Not Enough Information

Solution: $\frac{d}{dx}\left(\frac{1}{g(x)+2}\right) = \frac{-g'(x)}{(g(x)+2)^2} > 0$, so LEFT($n$) is an underestimate.

c. [3 points] If you were estimating $\int_0^1 \frac{dx}{g(x)+2}$ using TRAP(n), would your estimate be an overestimate, underestimate, or is there not enough information to tell? Circle your answer. You do not need to show your work or explain your answer.

Overestimate Underestimate Not Enough Information

Solution: $\frac{d^2}{dx^2}\left(\frac{1}{g(x)+2}\right) = \frac{-g''(x)(g(x)+2)^2 + 2(g'(x))^2(g(x)+2)}{(g(x)+2)^4} > 0$, so TRAP($n$) is an overestimate.