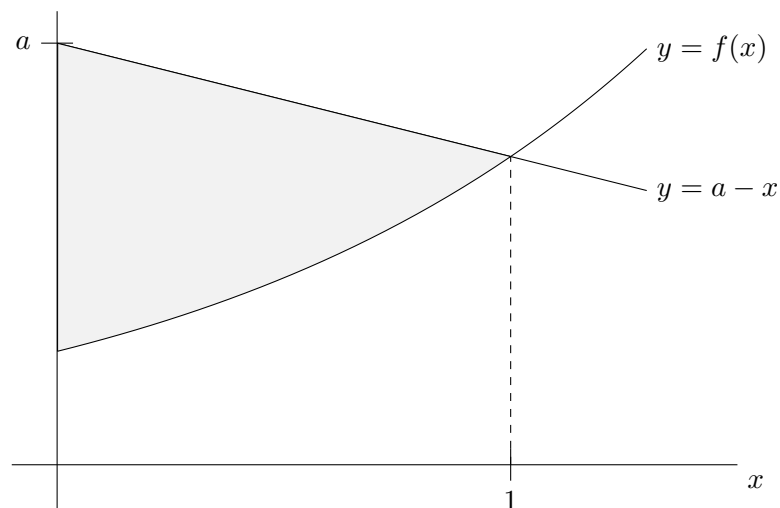


6. [7 points] A thin plate has the shape bounded by the curves  $y = f(x)$ ,  $y = a - x$  and the  $y$ -axis, for some positive constant  $a$ . The plate has a density of  $\delta(x) = 7 + x$  kg/m<sup>2</sup>.



- a. [4 points] Write an expression involving integrals that represents the total mass of the plate. Do not evaluate any integrals.

*Solution:* A thin vertical slice  $x$  m from the  $y$ -axis has a mass of  $((a - x) - f(x))\delta(x)\Delta x$ , so the total mass is  $\int_0^1 ((a - x) - f(x))(7 + x)dx$  kg.

- b. [3 points] Write an expression involving integrals that represents the  $x$ -coordinate of the center of mass of the plate. Do not evaluate any integrals.

*Solution:*  $\bar{x} = \frac{\int_0^1 x((a - x) - f(x))(7 + x)dx}{\int_0^1 ((a - x) - f(x))(7 + x)dx}$  meters.