1. [13 points] Suppose that f is a twice-differentiable, function that satisfies

$$f(0) = 1 \qquad f(2) = 2 \qquad f(4) = 4 \qquad f'(2) = 3$$

$$\int_0^2 f(x)\,dx = 5 \qquad \int_2^4 f(x)\,dx = 7.$$

Evaluate the following integrals.

a. [4 points] $\displaystyle\int_0^2 x f'(x)\,dx$

> *Solution:*
>
> $$\int_0^2 x f'(x)\,dx = x f(x)\big|_0^2 - \int_0^2 f(x)\,dx = -1.$$

b. [4 points] $\displaystyle\int_{\sqrt{2}}^2 x f'(x^2)\,dx$

> *Solution:*
>
> $$\int_{\sqrt{2}}^2 x f'(x^2)\,dx = \frac{1}{2}\int_2^4 f'(u)\,du = 1.$$

c. [5 points] $\displaystyle\int_0^2 x^3 f'(x^2)\,dx$

> *Solution:*
>
> $$\int_0^2 x^3 f'(x^2)\,dx = \frac{1}{2}\int_0^4 u f'(u)\,du = \frac{1}{2}\left(u f(u)\big|_0^4 - \int_0^4 f(u)\,du \right) = 2.$$