2. [16 points] The local sparrow population has been fluctuating unnaturally, and Raymond Green has five months of data to prove it. Let $S(t)$ denote the local sparrow population \textit{in thousands}, $t$ months after Green started collecting data. A graph of $S'(t)$, the rate of population growth, is below. Assume there are 2000 sparrows at $t = 1$.

\[ S'(t) \]

\[ \begin{array}{c|c|c|c|c|c} 
 t & 0 & 1 & 2 & 3 & 4 \\
 S'(t) & -1 & 0 & 2 & 0 & 1 \\
\end{array} \]

a. [1 point] At which $t$-value(s) is the sparrow population increasing the fastest?

\textit{Solution:} The population is increasing fastest at $t = 1$.

b. [3 points] What is the local sparrow population at $t = 0$, $t = 2$ and $t = 3$?

\textit{Solution:} The population is 500 at $t = 0$, 3000 at $t = 2$, and 2000 at $t = 3$.

c. [2 points] At which $t$-values is the population at its highest and lowest?

\textit{Solution:} The population is highest at $t = 5$ and lowest at $t = 0$. 
2 (continued). Recall that $S(t)$ is the local sparrow population in thousands, $t$ months after Green began collecting data.

\[ S'(t) \]

\[ \begin{array}{c|c|c|c|c|c|c|c|c}
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<td>0</td>
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<td>3</td>
<td>4</td>
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\end{array} \]

\[ S(t) \]

\[ \begin{array}{c|c|c|c|c|c|c|c}
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<tr>
<td>-1</td>
<td>0</td>
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<td>3</td>
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\end{array} \]

d. [10 points] Sketch a graph of $S(t)$ on the axes below, recalling that there are 2000 sparrows at $t = 1$. Label your vertical axis. Make sure that concavity and local extrema are clear.

Solution: