4. [15 points] For this problem, m is a differentiable function with $m'(x) > 0$ for all x. The following table gives some values of m.

x	0	1	2	3	4	5	6	7	8
$m(x)$	0	2	3	4	6	9	10	11	12

a. [3 points] What is the average value of $m'(x)$ on $[1, 7]$?

Solution: The average value is

$$\frac{1}{6}(m(7) - m(1)) = \frac{3}{2}.$$

b. [3 points] Use a left Riemann sum with 3 subdivisions to estimate $\int_2^8 m(x)\,dx$. Write out each term of your sum. Is this an overestimate or underestimate?

Solution: The left sum $2(3 + 6 + 10) = 38$ is an underestimate.

c. [3 points] Use a midpoint sum with 3 subdivisions to estimate $\int_0^{12} m^{-1}(y)\,dy$. Write out each term of your sum.

Solution: The correct sum is $4(1 + 4 + 6) = 44$.

d. [6 points] Consider the region bounded by the y-axis, the line $y = 12$ and the curve $y = m(x)$. Write an integral that gives the volume of the solid obtained by rotating this region about the y-axis. Use a right Riemann sum with 2 subdivisions to estimate your integral. Write out each term of your sum.

Solution: There are several possibilities. The shell method gives the volume as

$$2\pi \int_0^8 x(12 - m(x))\,dx,$$

where the associated right sum is $8\pi(4(12-6)+8(12-12)) = 192\pi$. The washer method gives the volume as

$$\pi \int_0^{12} (m^{-1}(y))^2\,dy,$$

and the associated right sum is $6\pi(4^2 + 8^2) = 480\pi$.