

9. [14 points] In a secret room at ShamCorp headquarters, there is a strangely-shaped transparent container filled with fluorescent purple liquid called “the key”. The key is in the shape of a solid with semicircular base of radius one meter, and with semicircular cross sections perpendicular to the straight side of the base. The key is suspended in the room with its semicircular cross sections parallel to the floor. The key has a volume of $\frac{\pi}{6} \text{ m}^3$, and the purple liquid has a density of 1500 kg/m^3 . The container that holds the purple liquid is infinitely thin and has no mass. For your reference, the gravitational constant is $g = 9.8 \text{ m/s}^2$.
- a. [7 points] One day, Dr. Durant orders Steph to move the key 2 meters higher. As soon as Steph begins to move the key straight up at a constant rate of 6 meters per minute, purple liquid starts leaking out of the key at a constant rate of $300\pi \text{ kg per minute}$. Write an expression involving integrals that gives the work done by Steph moving the key 2 meters higher as it’s leaking. Do not evaluate your integral.

Solution: The work done is

$$g \int_0^2 (250\pi - 50\pi h) dh \quad \text{J.}$$

- b. [7 points] Periodically, Steph has to do her least favorite job — emptying the key by pumping all of the purple liquid to a height of 3 meters above the top of the key. Write an expression involving integrals that gives the work done by Steph when she does this job, assuming the key is **full** when she starts. Do not evaluate your integral.

Solution: The work done is

$$\frac{1500g\pi}{8} \int_0^2 (1 - (h - 1)^2)(5 - h) dh \quad \text{J.}$$