10. [12 points] For each of the questions below, circle <u>all</u> of the available correct answers.
Circle "NONE OF THESE" if none of the available choices are correct.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [4 points] Suppose a function f and both its derivative f' and second derivative f'' are defined and continuous on the entire real line $(-\infty, \infty)$. Which of the following functions <u>must</u> be antiderivatives of the function $t^2 f'(t)$ on $(-\infty, \infty)$?

i. $\displaystyle\int_1^t 2y f''(y)\,dy$ ii. $\displaystyle 5 + \int_{-3}^t w^2 f'(w)\,dw$ iii. $\displaystyle 0.25\int_0^{2t} x^2 f'(0.5x)\,dx$

iv. $\displaystyle t^2 f(t) + \int_t^2 2x f(x)\,dx$ v. $\displaystyle f'(1) + \int_1^4 t^2 f'(t)\,dt$ vi. NONE OF THESE

b. [4 points] Suppose that g is a function that is continuous, negative, and decreasing on the interval $[-4, 4]$ and that n is a positive integer.
Consider the definite integral $\displaystyle\int_{-4}^4 g(x)\,dx$ and the four approximations of $\displaystyle\int_{-4}^4 g(x)\,dx$ given by RIGHT(n), LEFT(n), TRAP(n), MID(n).
Which of the following <u>could</u> be true about the relationships between these five numbers?

i. $\text{TRAP}(n) < \displaystyle\int_{-4}^4 g(x)\,dx$ ii. $\text{TRAP}(n) > \displaystyle\int_{-4}^4 g(x)\,dx$

iii. $\text{MID}(n) < \displaystyle\int_{-4}^4 g(x)\,dx$ iv. $\text{MID}(n) > \displaystyle\int_{-4}^4 g(x)\,dx$

v. $\text{RIGHT}(n) < \displaystyle\int_{-4}^4 g(x)\,dx < \text{LEFT}(n)$ vi. $\text{TRAP}(n) = \text{MID}(n)$

vii. $\text{LEFT}(n) < \displaystyle\int_{-4}^4 g(x)\,dx < \text{RIGHT}(n)$ viii. NONE OF THESE

c. [4 points] Suppose Q is a continuous function. A circular metal plate in the xy-plane with radius 10 cm has density $Q(r)$ grams per square centimeter at a distance of r centimeters from the center of the plate. Which of the following statements <u>must</u> be true about this plate?

i. The total mass of the plate is $100\pi \cdot Q(10)$ grams.

ii. The total mass of the plate is $\displaystyle\int_{-10}^{10} 2\pi r \cdot Q(r)\,dr$ grams.

iii. The mass, in grams, of a very thin horizontal slice of the plate of height Δy cm located y cm above the center of the plate is approximately $Q(y)$ times the area, in cm^2, of the slice.

iv. The total mass of the plate is $\displaystyle\int_0^{10} \pi r^2 \cdot Q(r)\,dr$ grams.

v. NONE OF THESE