1. [12 points] The table below gives several values of a decreasing, differentiable function $G$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G(x)$</td>
<td>$7$</td>
<td>$5$</td>
<td>$4$</td>
<td>$2$</td>
<td>$-2$</td>
<td>$-3$</td>
<td>$-6$</td>
<td>$-8$</td>
<td>$-9$</td>
</tr>
</tbody>
</table>

a. [4 points] Use the midpoint rule with 3 subintervals to estimate \( \int_{-4}^{2} (tG(t) + 4) \, dt \).

Carefully write out each of the terms involved in your estimate. You do not have to simplify. However, no variables or function names should appear in your answer.

**Solution:**

\[
\text{MID}(3) = 2 \cdot [(-3)G(-3) + 4] + 2 \cdot [(1)G(1) + 4] \\
= 2(-15 + 4) + 2(-2 + 4) + 2(-3 + 4) = -16
\]

So \( \int_{-4}^{2} (tG(t) + 4) \, dt \approx \text{MID}(3) = -16 \).

In parts b and c below, calculate the exact numerical value of the integral. If it is not possible to do so, write “NOT POSSIBLE”. *Show each step of your work clearly.*

b. [3 points] \( \int_{-2}^{4} 6G'(2y) \, dy \)

**Solution:** We use $w$-substitution with $w = 2y$. Then $dw = 2 \, dy$ so $dy = \frac{1}{2} \, dw$. Note that when $y = -2$ we have $w = -4$, and when $y = 2$, we have $w = 4$. (These values give the new limits of integration after substitution.) Substituting gives

\[
\int_{-2}^{4} 6G'(2y) \, dy = \int_{-4}^{4} 6G'(w) \frac{1}{2} \, dw = 3 \int_{-4}^{4} G'(w) \, dw = 3(G(4) - G(-4)) = 3(-9 - 7) = -48
\]

c. [5 points] \( \int_{0}^{3} \frac{G'(x)G(x)}{(2G(x) - 3)(G(x) + 1)} \, dx \)

**Solution:** The substitution $w = G(x)$ reduces the problem to an integral that can be computed using partial fractions. Notice that $w = -2$ when $x = 0$ and $w = -8$ when $x = 3$, so we have

\[
\int_{0}^{3} \frac{G'(x)G(x)}{(2G(x) - 3)(G(x) + 1)} \, dx = \int_{-2}^{-8} \frac{w}{(2w - 3)(w + 1)} \, dw = \int_{-2}^{-8} \left[ \frac{3}{10} \ln |2w - 3| + \frac{1}{5} \ln |w + 1| \right] \, dw \\
= \left[ \frac{3}{10} \ln |2w - 3| + \frac{1}{5} \ln |w + 1| \right]_{-2}^{-8} \\
= \left( \frac{3}{10} \ln(|-19|) + \frac{1}{5} \ln(|-7|) \right) - \left( \frac{3}{10} \ln(|-7|) + \frac{1}{5} \ln(|-1|) \right) \\
= \frac{3}{10} \ln(19) - \frac{1}{10} \ln(7) = \frac{1}{10} \ln(19^{2} / 7) = \frac{1}{10} \ln(6859 / 7).
\]