1. [12 points] The table below gives several values of a decreasing, differentiable function G.

x	-4	-3	-2	-1	0	1	2	3	4
$G(x)$	7	5	4	2	-2	-3	-6	-8	-9

a. [4 points] Use the midpoint rule with 3 subintervals to estimate $\displaystyle\int_{-4}^{2} \big(tG(t)+4\big)\,dt$.

Carefully write out each of the terms involved in your estimate.
You do **not** have to simplify. However, no variables or function names should appear in your answer.

Solution:

$$\begin{aligned}\text{MID}(3) &= 2\cdot[(-3)G(-3)+4] + 2\cdot[(-1)G(-1)+4] + 2\cdot[(1)G(1)+4]\\ &= 2(-15+4)+2(-2+4)+2(-3+4) = -16\end{aligned}$$

So $\displaystyle\int_{-4}^{2}\big(tG(t)+4\big)\,dt \approx \text{MID}(3) = \boxed{-16}$.

In parts **b** and **c** below, calculate the <u>exact</u> numerical value of the integral.
If it is not possible to do so, write "NOT POSSIBLE". *Show each step of your work clearly.*

b. [3 points] $\displaystyle\int_{-2}^{2} 6G'(2y)\,dy$

Solution: We use w-substitution with $w=2y$. Then $dw = 2\,dy$ so $dy = \frac{1}{2}dw$. Note that when $y=-2$ we have $w=-4$, and when $y=2$, we have $w=4$. (These values give the new limits of integration after substitution.) Substituting gives

$$\int_{-2}^{2} 6G'(2y)\,dy = \int_{-4}^{4} 6G'(w)\frac{1}{2}\,dw = 3\int_{-4}^{4} G'(w)dw = 3(G(4)-G(-4)) = 3(-9-7) = \boxed{-48}.$$

c. [5 points] $\displaystyle\int_{0}^{3} \frac{G'(x)\,G(x)}{(2G(x)-3)(G(x)+1)}\,dx$

Solution: The substitution $w=G(x)$ reduces the problem to an integral that can be computed using partial fractions. Notice that $w=-2$ when $x=0$ and $w=-8$ when $x=3$, so we have

$$\begin{aligned}\int_{0}^{3} \frac{G'(x)\,G(x)}{(2G(x)-3)(G(x)+1)}\,dx &= \int_{-2}^{-8} \frac{w}{(2w-3)(w+1)}\,dw = \int_{-2}^{-8} \frac{\frac{3}{5}}{2w-3} + \frac{\frac{1}{5}}{w+1}\,dw\\ &= \left[\frac{3}{10}\ln|2w-3| + \frac{1}{5}\ln|w+1|\right]\Bigg|_{-2}^{-8}\\ &= \left(\frac{3}{10}\ln(|-19|)+\frac{1}{5}\ln(|-7|)\right) - \left(\frac{3}{10}\ln(|-7|)+\frac{1}{5}\ln(|-1|)\right)\\ &= \frac{3}{10}\ln(19) - \frac{1}{10}\ln(7) = \frac{1}{10}\ln(19^3/7) = \boxed{\frac{1}{10}\ln(6859/7)}.\end{aligned}$$