3. [5 points] Suppose A is a differentiable function defined for all real numbers.

The function A has all of the following properties:

- A is an even function.
- $\int_{-2}^{2} A(x)\,dx = 5$.
- $A'(2) = 5$.
- The average value of A on the interval $[2,4]$ is $5/2$.

Based on the properties above, circle all of the statements below that <u>must</u> be true.
Circle "NONE OF THESE" if none of the statements must be true.
You must circle at least one choice to receive any credit for this problem. No credit will be awarded for unclear markings. No justification is necessary.

i. $A'(-2) = 5$.

For i: In fact, $A'(-2) = -5$. (To see this, think about graph symmetry or note that A' is odd since $A'(x) = \frac{d}{dx}(A(x)) = \frac{d}{dx}(A(-x)) = -A'(-x)$.)

ii. $\int_{0}^{2} A(x)\,dx = 5$.

For ii: Since A is even, $\int_{0}^{2} A(x)\,dx = \frac{1}{2}\int_{-2}^{2} A(x)\,dx = \frac{5}{2}$.

iii. $\int_{2}^{4} A(x)\,dx - \int_{-2}^{-4} A(x)\,dx = 0$.

For iii: $\int_{2}^{4} A(x)\,dx - \int_{-2}^{-4} A(x)\,dx = \int_{2}^{4} A(x)\,dx + \int_{-4}^{-2} A(x)\,dx = 2\int_{2}^{4} A(x)\,dx$ since A is even,

iv. $\boxed{\int_{-2}^{2} xA'(x)\,dx = 4A(2) - 5.}$

For iv: Use integration by parts.

v. $\boxed{\text{The function } R \text{ defined by } R(x) = \int_{-x}^{x} A'(t)\,dt \text{ \underline{must} be a constant function.}}$

For v: Note that $R(x) = A(x) - A(-x)$ so R must in fact be the constant <u>zero</u> function.

vi. NONE OF THESE