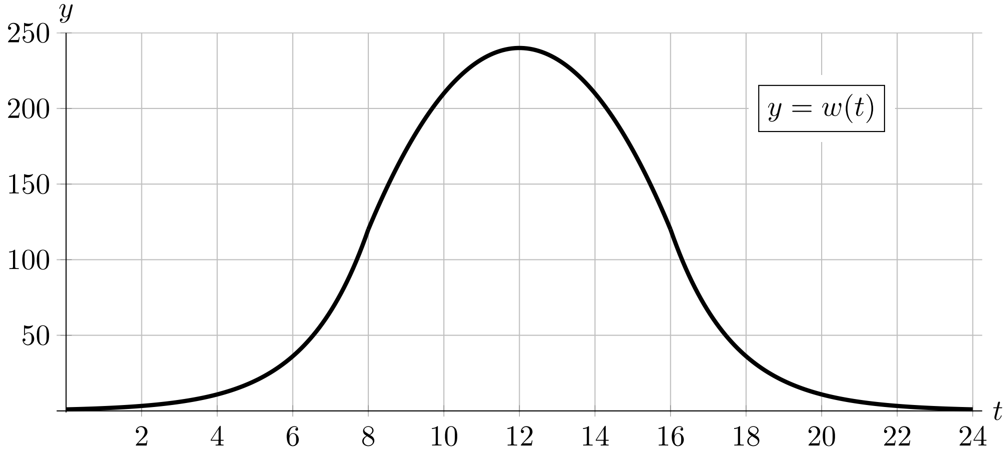
5. [10 points] Suppose that the function $w(t)$ shown in the graph below models the power, in kilowatts, that is harvested at a particular solar panel installation in northern Norway at time t, where t is measured in hours after midnight on a typical summer day.



Consider the function W defined by

$$W(x) = \int_{2x}^{2x+4} w(t)\,dt.$$

Be sure to show your work very carefully on all parts of this problem.

a. [3 points] Estimate $W(4)$. In the context of this problem, what are the units on $W(4)$?

Solution: Note that W gives the area beneath the graph of w during a four-hour interval. In particular, $W(4) = \int_8^{12} w(t)\,dt$ is the area beneath the graph of $w(t)$ between the hours of $t = 8$ and $t = 12$. Estimating this integral (or estimating the area geometrically) gives $W(4) \approx 800$. The units on W are kilowatt·hours.

Answer: $W(4) \approx$ 800 **Units:** **kilowatt·hours**

b. [4 points] Estimate $W'(4)$. In the context of this problem, what are the units on $W'(4)$?

Solution: By the (first or second) Fundamental Theorem of Calculus together with the Chain Rule, we have

$$W'(x) = w(2x+4)\cdot(2) - w(2x)\cdot(2).$$

Substituting $x = 4$ gives

$$W'(4) = w(12)\cdot(2) - w(8)\cdot(2) = (240)\cdot(2) - (120)\cdot(2) = 240.$$

The units on W' are (kilowatts·hours)/(hours)=kilowatts.

Answer: $W'(4) \approx$ 240 **Units:** **kilowatts**

c. [3 points] Estimate the value(s) of x at which $W(x)$ attains its maximum value on the interval $0 \le x \le 8$. If there are no such values, explain why.

Solution: The function $W(x)$ gives the area beneath the graph of $w(t)$ during the four-hour interval between $t = 2x$ and $t = 2x + 4$. By inspecting the graph, one sees that this area is largest between the hours of $t = 10$ and $t = 14$, corresponding to $x = 5$. That is, $W(5)$ gives this maximal area, so $W(x)$ attains its maximum value at $x = 5$.