5. [10 points] Suppose that the function \( w(t) \) shown in the graph below models the power, in kilowatts, that is harvested at a particular solar panel installation in northern Norway at time \( t \), where \( t \) is measured in hours after midnight on a typical summer day.

Consider the function \( W \) defined by

\[
W(x) = \int_{2x}^{2x+4} w(t) \, dt.
\]

Be sure to show your work very carefully on all parts of this problem.

a. [3 points] Estimate \( W(4) \). In the context of this problem, what are the units on \( W(4) \)?

Solution: Note that \( W \) gives the area beneath the graph of \( w \) during a four-hour interval. In particular, \( W(4) = \int_{8}^{12} w(t) \, dt \) is the area beneath the graph of \( w(t) \) between the hours of \( t = 8 \) and \( t = 12 \). Estimating this integral (or estimating the area geometrically) gives \( W(4) \approx 800 \). The units on \( W \) are kilowatt-hours.

Answer: \( W(4) \approx 800 \) Units: kilowatt-hours

b. [4 points] Estimate \( W'(4) \). In the context of this problem, what are the units on \( W'(4) \)?

Solution: By the (first or second) Fundamental Theorem of Calculus together with the Chain Rule, we have

\[
W'(x) = w(2x + 4) \cdot (2) - w(2x) \cdot (2).
\]

Substituting \( x = 4 \) gives

\[
W'(4) = w(12) \cdot (2) - w(8) \cdot (2) = (240) \cdot (2) - (120) \cdot (2) = 240.
\]

The units on \( W' \) are (kilowatts-hours)/(hours)=kilowatts.

Answer: \( W'(4) \approx 240 \) Units: kilowatts

c. [3 points] Estimate the value(s) of \( x \) at which \( W(x) \) attains its maximum value on the interval \( 0 \leq x \leq 8 \). If there are no such values, explain why.

Solution: The function \( W(x) \) gives the area beneath the graph of \( w(t) \) during the four-hour interval between \( t = 2x \) and \( t = 2x+4 \). By inspecting the graph, one sees that this area is largest between the hours of \( t = 10 \) and \( t = 14 \), corresponding to \( x = 5 \). That is, \( W(5) \) gives this maximal area, so \( W(x) \) attains its maximum value at \( x = 5 \).