7. [12 points] Maria has a toy car that drives around her flat backyard. She describes the path of the car by typing a pair of parametric equations into a computer navigation system. The computer controller uses \( x \)- and \( y \)-coordinates, where the units of the axes are meters, the point where Maria will be standing corresponds to the origin \((x, y) = (0, 0)\), the positive \(y\)-axis points north, and the positive \(x\)-axis points east. The car’s battery will only last 60 minutes, so Maria sets the domain of each of her parametric equations to be \(0 \leq t \leq 60\), where \(t\) is measured in minutes.

Maria enters the parametric equations \(x = f(t)\) and \(y = g(t)\) where \(f\) and \(g\) are the functions shown in the graphs below.

![Graphs of x = f(t) and y = g(t)](image)

**a.** [3 points] The tangent line to the graph of \(y = g(t)\) at the point \(t = 40\) has equation \(y - 10 = -2(t - 40)\). (This is the dashed line shown in the \(ty\)-plane above.) Use this information to compute the instantaneous speed of Maria’s car at time \(t = 40\). Be sure to show your work clearly.

**Solution:** Note that \(\frac{dx}{dt} = f'(t)\) and \(\frac{dy}{dt} = g'(t)\). The slope of the given tangent line at \(t = 40\) is the value of \(g'(40)\), so \(g'(40) = -2\). By examining the graph of \(x = f(t)\) at \(t = 40\), we also see that \(f'(40) = 0\). The instantaneous speed of Maria’s car at time \(t = 40\) is therefore \(\sqrt{(f'(40))^2 + (g'(40))^2} = \sqrt{0^2 + (-2)^2} = 2\) meters per minute.

**b.** [2 points] At time \(t = 0\), the car starts at Maria’s location. Approximately how many meters away from Maria will the car be at time \(t = 60\) (when it will run out of power)? Circle the one best estimate from among the choices below.

- 0 m
- 150 m
- 300 m
- 450 m
- 600 m
- 750 m

**c.** [3 points] At which of the times listed below is the slope of Maria car’s path in the \(xy\)-plane the least (most negative)? Circle the one best answer from among the choices below.

- \(t = 15\)
- \(t = 20\)
- \(t = 28\)
- \(t = 32\)
- \(t = 38\)

**d.** [4 points] Maria’s friend William programs his car to move according to the parametric equations \(x = \int_0^t f(s) \, ds\) and \(y = \int_0^t g(s) \, ds\) where \(f\) and \(g\) are the functions shown in the graphs above. Compute the instantaneous speed of William’s car at time \(t = 20\). Be sure to show your work clearly.

**Solution:** By the Second Fundamental Theorem of Calculus, we have \(\frac{dx}{dt} = f(t)\) and \(\frac{dy}{dt} = g(t)\). By examining the graphs, we therefore see that \(\frac{dx}{dt}\big|_{t=20} = f(20) = 20\) and \(\frac{dy}{dt}\big|_{t=20} = g(20) = 10\). Hence, the instantaneous speed of William’s car at time \(t = 20\) is \(\sqrt{(20)^2 + (10)^2} = \sqrt{500} = 10\sqrt{5}\) meters per minute.