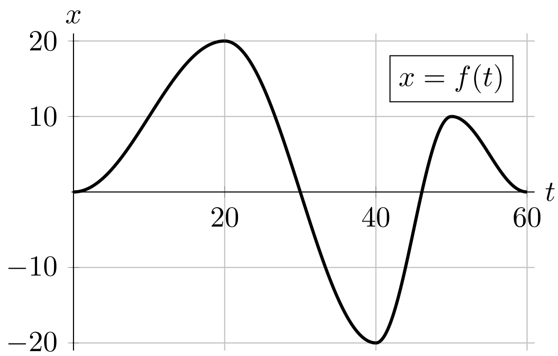
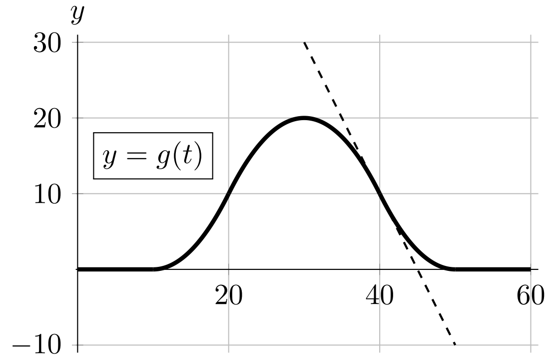
7. [12 points] Maria has a toy car that drives around her flat backyard. She describes the path of the car by typing a pair of parametric equations into a computer navigation system.
The computer controller uses x- and y-coordinates, where the units of the axes are meters, the point where Maria will be standing corresponds to the origin $(x, y) = (0, 0)$, the positive y-axis points north, and the positive x-axis points east. The car's battery will only last 60 minutes, so Maria sets the domain of each of her parametric equations to be $0 \leq t \leq 60$, where t is measured in minutes.
Maria enters the parametric equations $\boxed{x = f(t) \quad \text{and} \quad y = g(t)}$
where f and g are the functions shown in the graphs below.





a. [3 points] The tangent line to the graph of $y = g(t)$ at the point $t = 40$ has equation $y - 10 = -2(t - 40)$. (This is the dashed line shown in the ty-plane above.) Use this information to compute the instantaneous speed of Maria's car at time $t = 40$.
Be sure to show your work clearly.

Solution: Note that $\frac{dx}{dt} = f'(t)$ and $\frac{dy}{dt} = g'(t)$. The slope of the given tangent line at $t = 40$ is the value of $g'(40)$, so $g'(40) = -2$. By examining the graph of $x = f(t)$ at $t = 40$, we also see that $f'(40) = 0$. The instantaneous speed of Maria's car at time $t = 40$ is therefore $\sqrt{(f'(40))^2 + (g'(40))^2} = \sqrt{0^2 + (-2)^2} = \boxed{\text{2 meters per minute.}}$

b. [2 points] At time $t = 0$, the car starts at Maria's location. Approximately how many meters away from Maria will the car be at time $t = 60$ (when it will run out of power)? Circle the <u>one</u> best estimate from among the choices below.

$\boxed{\text{0 m}}$ 150 m 300 m 450 m 600 m 750 m

c. [3 points] At which of the times listed below is the slope of Maria car's path in the xy-plane the least (most negative)? Circle the <u>one</u> best answer from among the choices below.

$t = 15$ $t = 20$ $\boxed{t = 28}$ $t = 32$ $t = 38$

d. [4 points] Maria's friend William programs his car to move according to the parametric equations

$$x = \int_0^t f(s)\, ds \qquad \text{and} \qquad y = \int_0^t g(s)\, ds$$

where f and g are the functions shown in the graphs above. Compute the instantaneous speed of William's car at time $t = 20$. Be sure to show your work clearly.

Solution: By the Second Fundamental Theorem of Calculus, we have $\frac{dx}{dt} = f(t)$ and $\frac{dy}{dt} = g(t)$. By examining the graphs, we therefore see that $\frac{dx}{dt}\Big|_{t=20} = f(20) = 20$ and $\frac{dy}{dt}\Big|_{t=20} = g(20) = 10$. Hence, the instantaneous speed of William's car at time $t = 20$ is $\sqrt{(20)^2 + (10)^2} = \sqrt{500} = \boxed{10\sqrt{5} \text{ meters per minute.}}$