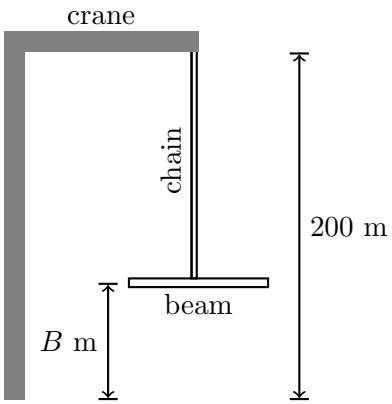
9. [8 points]
During the construction of a skyscraper, a 200 meter tall crane lifts a steel beam from the ground to a height of 175 meters. The steel beam has a mass of 50 kilograms. The crane has a chain that is also made of steel, and the chain has a mass of 15 kilograms per meter. The total length of the chain is 200 meters, but as the beam is lifted, the crane no longer needs to lift any of the chain that has already been "reeled in", i.e. has already reached the top of the crane.



For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2 .

a. Write an expression in terms of B that gives the total mass, in kilograms, of the steel beam together with the chain that has not yet been reeled in at the moment that the steel beam is B meters above the ground.

Solution: If the steel beam is B meters above the ground, that means that a length of B meters of chain has been reeled in, so $(200-B)$ meters of chain remains. The mass of this remaining chain is (15 kilograms per meter) $\cdot$ $(200 - B$ meters$) = 3000 - 15B$ kilograms. We add to this the mass of the steel beam to find a total mass of

$$\text{Mass} = 50 + 3000 - 15B \text{ kilograms.}$$

b. Assuming ΔB is very small but positive, write an expression in terms of B that approximates the work done by the crane in lifting the steel beam up ΔB meters starting from a height of B meters above the ground. Assume that the weight of the chain being lifted is constant over this very short distance. Include units.

Solution: The force due to gravity is the weight. At the moment the steel beam is B meters above the ground, the weight of the beam together with the chain that has not yet been reeled in is

$$\text{Weight} = (\text{mass})(g) = (3050 - 15B \text{ kilograms})(9.8 \text{ m/s}^2) = 29890 - 147B \text{ Newtons.}$$

The work to lift the steel beam over the small distance ΔB is then approximately

$$(\text{Force}) \cdot \Delta B = (29890 - 147B)\Delta B \text{ Joules.}$$

c. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work that must be done by the crane in order to lift the steel beam from the ground to a height of 175 meters. Include units.

Solution: Using our answer from part **b.** above, summing over the entire path of the beam, and taking the limit as ΔB approaches 0, we find that the total work is

$$\text{Work} = \int_0^{175} g(3050 - 15B)\, dB = \int_0^{175} (29890 - 147B)\, dB \text{ Joules.}$$