1. [12 points] The table below gives several values of a differentiable function f such that f' is also differentiable and f'' is continuous.

x	-3	-2	-1	0	1	2	3
$f(x)$	14	20	4	11	24	5	8
$f'(x)$	3	-4	-6	2	5	-3	4

For each of the following, calculate the exact numerical value of the integral. If it is not possible to do so based on the information provided, write "NOT POSSIBLE" and clearly indicate why it is not possible. Show your work.
Note that no variables or function names (such as f, f', or f'') should appear in your answers.

a. [3 points] $\int_0^1 f'(2x)\,dx$

let $w = 2x$
$dw = 2dx$
$x = 0 \Rightarrow w = 0$
$x = 1 \Rightarrow w = 2$

$= \int_0^2 f'(w)\cdot\frac{dw}{2}$

$= \frac{1}{2}\left[f(w)\right]_0^2 = \frac{1}{2}\left[f(2) - f(0)\right] = \frac{1}{2}\left[5 - 11\right]$

Answer: -3

b. [3 points] $\int_2^3 sf''(s)\,ds = \int_2^3 uv' = uv\Big|_2^3 - \int_2^3 u'v = sf'(s)\Big|_2^3 - \int_2^3 (1)(f'(s))\,ds$

Parts:
Let $u = s$ $\quad v' = f''(s)$
$u' = 1$ $\quad v = f'(s)$

$= sf'(s) - f(s)\Big|_2^3 = \left[3f'(3) - f(3)\right] - \left[2f'(2) - f(2)\right]$

$= \left[3(4) - 8\right] - \left[2(-3) - 5\right] = \left[12 - 8\right] - \left[-6 - 5\right] = 4 + 11$

Answer: 15

c. [3 points] $\int_{-2}^{-1} q\cdot\left[\frac{d}{dq}\left(f'(q)e^{f(q)}\right)\right] dq = \int_{-2}^{-1} uv' = \underbrace{uv\Big|_{-2}^{-1}}_{①} - \underbrace{\int_{-2}^{-1} u'v}_{②}$.

Parts:
Let $u = q$ $\quad v' = \frac{d}{dq}\left(f'(q)e^{f(q)}\right)$
$u' = 1$ $\quad v = f'(q)e^{f(q)}$

① $uv\Big|_{-2}^{-1} = qf'(q)e^{f(q)}\Big|_{-2}^{-1} = (-1)f'(-1)e^{f(-1)} - (-2)f'(-2)e^{f(-2)}$
$= (-1)(-6)e^4 - (-2)(-4)e^{20} = 6e^4 - 8e^{20}$

② $\int_{-2}^{-1} u'v = \int_{-2}^{-1} (1)f'(q)e^{f(q)}\,dq$ $\quad$ let $w = f(q)$, $dw = f'(q)\,dq$, $q = -2 \Rightarrow w = f(-2) = 20$, $q = -1 \Rightarrow w = f(-1) = 4$

$= \int_{20}^{4} e^w\,dw = e^w\Big|_{20}^{4} = e^4 - e^{20}$

So we have ① − ② $= (6e^4 - 8e^{20}) - (e^4 - e^{20})$

Answer: $5e^4 - 7e^{20}$

d. [3 points] $\int_{-1}^{1} f'(y)\cdot f''(f(y))\,dy$

Let $w = f(y)$
$dw = f'(y)\,dy$
$y = -1 \Rightarrow w = f(-1) = 4$
$y = 1 \Rightarrow w = f(1) = 24$

$= \int_4^{24} f''(w)\,dw = f'(w)\Big|_4^{24} = f'(24) - f'(4)$.

But we don't know $f'(24)$ and $f'(4)$, so we can't go any further.

Answer: NOT POSSIBLE