3. [8 points]
Consider a tent that is 50 meters tall whose base is a regular hexagon (i.e. a 6-sided polygon with equal length sides and equal angles) and whose horizontal cross-sections are also regular hexagons.
(See figure on the right.)
Suppose the perimeter of the base is 72 meters.
Let $P(y)$ be the perimeter, in meters, of a horizontal cross section $y$ meters above the ground.

a. [2 points] It turns out that $P(y)$ is a linear function of the variable $y$.
   (You do not need to verify this.) Find a formula for $P(y)$.

   \[
   \begin{array}{c|c}
   y & P(y) \\
   \hline
   0 & 72 \\
   50 & 0 \\
   \end{array}
   \]

   \[
   \text{Slope} = \frac{0 - 72}{50 - 0} = -1.44
   \]

   \[
   \text{Answer: } P(y) = 72 - 1.44y
   \]

b. [3 points] The area of a regular hexagon with perimeter $p$ is equal to $\frac{\sqrt{3}}{24}p^2$.

   Write an expression that gives the approximate volume, in cubic meters, of a horizontal slice of the region inside the tent that is $\Delta y$ meters thick and $y$ meters above the ground. (Assume here that $\Delta y$ is small but positive.) Your expression should not involve any integrals.

   \[
   \text{Area of slice} = \frac{\sqrt{3}}{24} P(y)^2 = \frac{\sqrt{3}}{24} \left(72 - 1.44y\right)^2
   \]

   \[
   \text{Volume of slice} = \frac{\sqrt{3}}{24} \left(72 - 1.44y\right)^2 \Delta y
   \]

   \[
   \text{Answer: Volume of slice} \approx \frac{\sqrt{3}}{24} \left(72 - 1.44y\right)^2 \Delta y
   \]

c. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the total volume, in cubic meters, inside the tent.

   \[
   \int_0^{50} \frac{\sqrt{3}}{24} \left(72 - 1.44y\right)^2 dy
   \]

   \[
   \text{Answer: Volume} = \int_0^{50} \frac{\sqrt{3}}{24} \left(72 - 1.44y\right)^2 dy = 36000\sqrt{3} \text{ m}^3
   \]