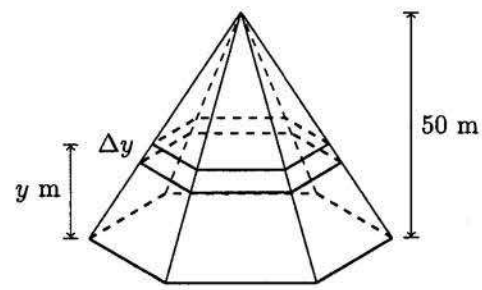
3. [8 points]
Consider a tent that is 50 meters tall whose base is a regular hexagon (i.e. a 6-sided polygon with equal length sides and equal angles) and whose horizontal cross-sections are also regular hexagons.
(See figure on the right.)
Suppose the perimeter of the base is 72 meters.
Let $P(y)$ be the perimeter, in meters, of a horizontal cross section y meters above the ground.



a. [2 points] It turns out that $P(y)$ is a linear function of the variable y. (You do not need to verify this.) Find a formula for $P(y)$.

y	$P(y)$
0	72
50	0

Slope $= \frac{0-72}{50-0} = -1.44$

Answer: $P(y) =$ $72 - 1.44y$

b. [3 points] The area of a regular hexagon with perimeter p is equal to $\frac{\sqrt{3}}{24}p^2$.

Write an expression that gives the approximate volume, in cubic meters, of a horizontal slice of the region inside the tent that is Δy meters thick and y meters above the ground. (Assume here that Δy is small but positive.) Your expression should <u>not</u> involve any integrals.

area of slice $= \frac{\sqrt{3}}{24} P(y)^2 = \frac{\sqrt{3}}{24}(72-1.44y)^2$

Volume of slice $= \frac{\sqrt{3}}{24}(72-1.44y)^2 \Delta y$

Answer: Volume of slice $\approx$ $\frac{\sqrt{3}}{24}(72-1.44y)^2 \Delta y$

c. [3 points] Write, but do <u>not</u> evaluate, an expression involving one or more integrals that gives the total volume, in cubic meters, inside the tent.

Answer: Volume $=$ $\boxed{\int_0^{50} \frac{\sqrt{3}}{24}(72-1.44y)^2\,dy} = 3600\sqrt{3}\ m^3$