4. [12 points] For each of the questions below, circle <u>all</u> of the available correct answers.
Circle "NONE OF THESE" if none of the available choices are correct.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Suppose f and g are continuous functions defined for all real numbers. Which of the following <u>must</u> be true?

i. The average value of the sum of f and g over the interval $[-5,5]$ is equal to the sum of the average value of f over $[-5,5]$ and the average value of g over $[-5,5]$.

ii. $\int_{-5}^{5}(3+f(x))\,dx = 3+\int_{-5}^{5} f(x)\,dx$

iii. $\int f'(x^2)\,dx = \dfrac{f(x^2)}{2x}+C$

iv. $\int_{-5}^{5} f(x)\cdot g(x)\,dx = \int_{-5}^{5} f(x)\,dx \cdot \int_{-5}^{5} g(x)\,dx$

v. NONE OF THESE

b. [3 points] Suppose that W is a function that is continuous and positive on the interval $[0,1]$. Consider the the approximations RIGHT(100), LEFT(100), and MID(100) of the definite integral $\int_0^1 W(x)\,dx$. Which of the following <u>must</u> be true?

Unless you know something about W (e.g. increasing, decreasing, concave up or concave down), all Riemann sum estimates could be way off, in either direction.

i. $\text{LEFT}(100) \le \int_0^1 W(x)\,dx$

ii. $\text{RIGHT}(100) \le \int_0^1 W(x)\,dx$

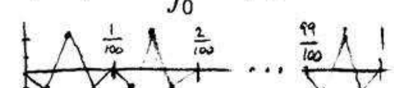
iii. $\text{LEFT}(100) \ge \int_0^1 W(x)\,dx$

iv. $\text{RIGHT}(100) \ge \int_0^1 W(x)\,dx$

v. MID(100) is at least as close to $\int_0^1 W(x)\,dx$ as RIGHT(100) is to $\int_0^1 W(x)\,dx$.

vi. NONE OF THESE

Counterexample:



RIGHT(100) = $\int_0^1 W(x)dx = 0$, but MID(100) > 0.

c. [3 points] Let $Q(x)=\int_1^x \ln(t)\,dt$. Then which of the following <u>must</u> be true?

i. $x = \dfrac{d}{dx}\left[\int_1^{e^x} \ln(t)\,dt\right]$

ii. $\dfrac{d}{dp}\left[Q(4+\sin(p))\right] = \cos(p)\left[\ln(4+\sin(p))\right]$

iii. $Q'(x)=\ln(x)$

iv. $\dfrac{d}{dr}\left[\int_1^{1+r^2} \ln(t)\,dt\right] = \dfrac{d}{dr}\left[\int_1^{r} \ln(t^2+1)\,dt\right]$

v. NONE OF THESE

d. [3 points] Let $g(x)$ be a differentiable function that is decreasing and concave up on the interval $[0,1]$. Suppose $G(x)$ is an antiderivative of $g(x)$. Which of the following <u>must</u> be true?

i. $G(x)\le 0$ on $[0,1]$.

ii. $G(x)$ is increasing on $[0,1]$.

iii. $G(x)$ is concave down on $(0,1)$.

iv. $G(x)$ has no inflection points in $(0,1)$.

v. NONE OF THESE