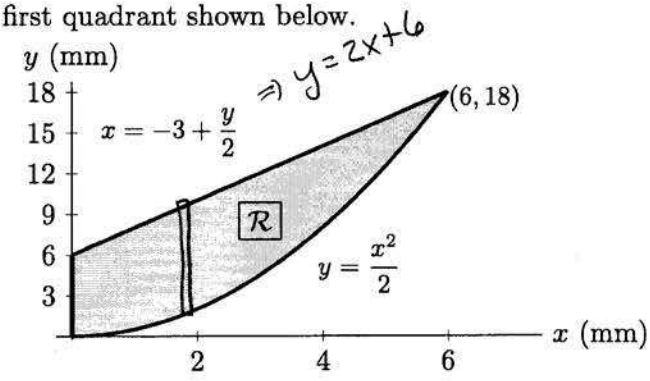
6. [12 points] Let $\mathcal{R}$ be the shaded region in the first quadrant shown below.

The region $\mathcal{R}$ is bounded by:

- the y-axis,
- the graph of $y = \dfrac{x^2}{2}$, and
- the graph of $x = -3 + \dfrac{y}{2}$. $\boxed{y = 2x+6}$

The units on both axes are millimeters (mm).



a. [4 points] Write, but do NOT evaluate, an expression involving one or more integrals that gives the volume, in mm^3, of the solid whose base is the region $\mathcal{R}$ and whose cross-sections perpendicular to the x-axis are squares.

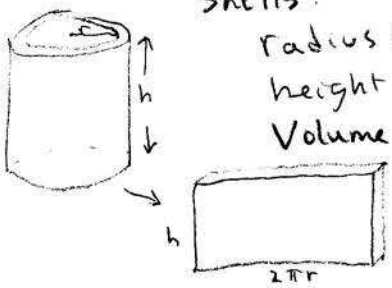
Side of slice $= (2x+6) - \dfrac{x^2}{2} = -\frac{1}{2}x^2 + 2x + 6$

area of slice $= \left(-\frac{1}{2}x^2+2x+6\right)^2$

Volume of slice $= \left(-\frac{1}{2}x^2+2x+6\right)^2 \Delta x$

Answer: Volume = $\displaystyle\int_0^6 \left(-\frac{1}{2}x^2+2x+6\right)^2 dx$

b. [4 points] Write, but do NOT evaluate, an expression involving one or more integrals that gives the volume, in mm^3, of the solid formed by rotating the region $\mathcal{R}$ around the y-axis.



Shells:

radius of shell $= x$

height of shell $= -\frac{1}{2}x^2 + 2x + 6$

Volume of shell $= 2\pi r h \Delta x = 2\pi x\left(-\frac{1}{2}x^2+2x+6\right)\Delta x$

Answer: Volume = $\displaystyle\int_0^6 2\pi x\left(-\frac{1}{2}x^2+2x+6\right) dx$

c. [4 points] Write, but do NOT evaluate, an expression involving one or more integrals that gives the mass, in grams, of a thin plate in the shape of the region $\mathcal{R}$ that has mass density given by $\delta(x) = (1+x^2)$ g/mm^2.

Area of slice $= \left(-\frac{1}{2}x^2 + 2x + 6\right)\Delta x$

mass of slice $=$ (Area)(density) $= \left(-\frac{1}{2}x^2+2x+6\right)(1+x^2)\Delta x$

Answer: Mass = $\displaystyle\int_0^6 \left(-\frac{1}{2}x^2 + 2x + 6\right)(1+x^2) dx$