7. [12 points] Note that the problems on this page do not depend on each other.

a. [4 points] Suppose $F(x)$ is an antiderivative of $f(x) = e^{-x^2}$ such that $F(2) = 10$. Write an integral expression for the function $F(x)$. (Your expression should not involve the letters f or F.) Remember to be careful with notation.

Answer: $F(x) =$ $10 + \int_2^x e^{-t^2}\,dt$

b. [4 points] Suppose $H(x)$ is an antiderivative of $h(x) = \sin(x^2)$. Write an expression for the average value of $h(x)$ on the interval $[-1, 1]$. Your expression should not involve any integrals but may involve function names.

$$\text{avg Value} = \frac{1}{1-(-1)}\int_{-1}^{1} h(x)\,dx$$

Answer: Average Value = $\frac{1}{2}\left[H(1) - H(-1)\right]$

c. [4 points] Suppose $G(x)$ is an antiderivative of $g(x) = \sqrt{x^4 - 1}$ for $x > 1$. Find the arc length of the graph of $G(x)$ from $x = 2$ to $x = 3$. Show your work. You may use your calculator to evaluate any integrals. Give the exact answer or round to two decimal places.

$$\text{arclen} = \int_a^b \sqrt{1 + G'(x)^2}\,dx = \int_2^3 \sqrt{1 + (x^4 - 1)}\,dx$$

$$= \int_2^3 x^2\,dx = \frac{1}{3}x^3\Big|_2^3 = \frac{1}{3}\left[3^3 - 2^3\right]$$

Answer: Arc Length = $19/3$