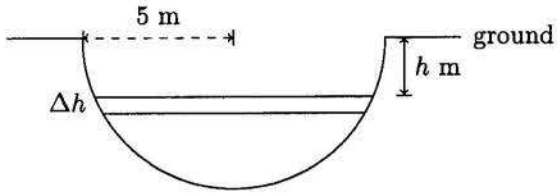
8. [12 points]

Alicia is building a pond in her backyard. The pond will be in the shape of hemisphere with radius 5 meters. A side view of the hole for the pond is shown in the figure on the right.
Note: The curved region shown is a semicircle of radius 5 meters, and cross-sections of the hole parallel to the ground are circles.



Alicia discovers that the density (in kg/m^3) of the dirt in her yard is given by the function $\rho(h) = 1.5 + (h-1)^3$ where h is distance (in meters) below ground. In this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2.

a. [4 points] Write an expression that gives the approximate mass of a horizontal slice of dirt with thickness Δh meters that is h meters below the ground. See diagram. (Assume that Δh is small but positive.) Your expression should not involve any integrals.

$r^2 + h^2 = 5^2$

radius of slice $= \sqrt{25-h^2}$ m

volume of slice $= \pi r^2 \Delta h = \pi(25-h^2)\Delta h$ m^3

density of slice $= \rho(h) = 1.5 + (h-1)^3$ kg/m^3

mass of slice = (volume)(density) =

Answer: Mass of slice $\approx$ $\pi(25-h^2)(1.5+(h-1)^3)\Delta h$ kg

b. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the mass (in kg) of the dirt Alicia removes in order to create the hole for her pond.

Answer: Mass = $\int_{h=0}^{5} \pi(25-h^2)(1.5+(h-1)^3)\,dh$ kg

c. [5 points] As Alicia digs, she lifts the dirt 1 meter above the ground to put it into the back of a truck. Write, but do not evaluate, an expression involving one or more integrals that gives the work Alicia does to remove all the dirt from the hole for her pond.

weight of slice = (mass)(acceleration due to gravity)
$= 9.8\pi(25-h^2)(1.5+(h-1)^3)\Delta h$ N

dist to lift slice $= h+1$ m

work to lift slice = (weight)(distance)

Answer: Work = $\int_0^5 9.8\pi(h+1)(25-h^2)(1.5+(h-1)^3)\,dh$ joules (include units)