9. [13 points] Simba is currently practicing his tennis swing by hitting a ball against a wall. During his first attempt he only managed to hit the ball 5 times. Each time he practices, he manages to hit the ball more so that in his nth attempt he hits the ball $2n^2 + 2n + 1$ times. After 10 attempts he stops and makes the following claim:

Simba's Claim: I must have the hit ball at least 800 times!

The total number of hits in his first 10 attempts is given by the following sum:

$$H = \sum_{n=1}^{10} (2n^2 + 2n + 1)$$

a. [3 points] Find a definite integral for which RIGHT(10) $= H$.

if $\Delta x = 1$, $H = \sum_{n=1}^{10} (2x^2 + 2x + 1)\Delta x$, so

Answer: $\int_0^{10} (2x^2 + 2x + 1)\, dx$

b. [3 points] Using only your integral from part (a), is it possible to evaluate Simba's claim? If so, evaluate his claim using this integral.
Note: To earn credit, clear justification based on your integral must be provided for the answer you circle.

Answer: Based only on your integral from part (a), we can conclude that Simba

Circle your answer.

Hit the ball at least 800 times Did not hit the ball 800 times Not possible to tell

Justification:

Since $2x^2 + 2x + 1$ is increasing on $[0, 10]$ (because its derivative, $4x + 2$, is positive),

$$\text{RIGHT}(10) \geq \int_0^{10} (2x^2 + 2x + 1)\, dx = \frac{2}{3}x^3 + x^2 + x \Big|_0^{10} = \left[\left(\frac{2}{3}(1000) + 100 + 10\right) - 0\right] = 776\frac{2}{3}$$

We know he hit it at least 777 times, but he might not have made it to 800.

This problem continues on the next page.

This is a continuation of the problem from the previous page.
For your convenience, a reminder of the setup is included here.

Simba is currently practicing his tennis swing by hitting a ball against a wall. After 10 attempts he stops and makes the following claim:

Simba's Claim: I must have the hit ball at least 800 times!

The total number of hits in his first 10 attempts is given by: $H = \sum_{n=1}^{10} (2n^2 + 2n + 1)$.

c. [4 points]Note that $\sum_{n=1}^{10} (2n^2 + 2n + 1) = 2 \cdot \sum_{n=1}^{10} \left[\frac{1}{2}\left(n^2 + (n+1)^2\right)\right]$.
(You do not need to verify this.)

Use the resulting formula $H = 2 \cdot \sum_{n=1}^{10} \left[\frac{1}{2}\left(n^2 + (n+1)^2\right)\right]$

to find a definite integral for which TRAP(10) $= H$.

Answer: $\int_1^{11} 2x^2\,dx$

d. [3 points] Using only your integral from part (c) is it possible to evaluate Simba's claim? If so, evaluate his claim using this integral.
Note: To earn credit, clear justification based on your integral must be provided for the answer you circle.

Answer: Based only on your integral from part (c), we can conclude that Simba

Circle your answer.

Hit the ball at least 800 times Did not hit the ball 800 times Not possible to tell

Justification:

Because $2x^2$ is concave up, TRAP(10) is an over estimate for the integral, which means the integral is an underestimate of the sum. The integral is

$$\int_1^{11} 2x^2\,dx = \frac{2}{3}x^3\Big|_1^{11} = \frac{2}{3}(11)^3 - \frac{2}{3}(1)^3 = 886\frac{2}{3}.$$

So the sum is at least that much.
(Actual value of sum is 890. TRAP is much closer than RIGHT!)