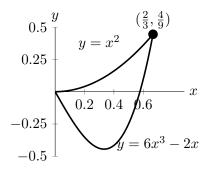
- **3.** [9 points] Scientists are studying a cave. The inside of the cave can be modeled as a solid in the following way:
 - the base is a region bounded by the curves $y = 6x^3 2x$ and $y = x^2$
 - cross-sections perpendicular to the x-axis are squares
 - the cave's entrance is at the origin
 - x is measured in miles east of the entrance, and y is measured in miles north of the entrance



- **a**. [6 points] The scientists want to know the volume of the cave.
 - (i) Write, but **do not evaluate**, an expression that gives the approximate volume, in cubic miles, of a vertical slice of the cave that is Δx miles thick and x miles east from the entrance of the cave.
 - Answer: Volume of slice $\approx (x^2 (6x^3 2x))^2 \Delta x$
 - (ii) Write, but do not evaluate, an expression involving one or more integrals that gives the total volume, in cubic miles, of the cave.

Answer:

 $\int_{0}^{2/3} (x^2 - (6x^3 - 2x))^2 dx$ b. [3 points] Salamanders live on the floor of the cave at a density that depends on the x distance from the entrance. Let $\delta(x)$ be their population density, in salamanders per square mile. Write, but do not evaluate, an expression involving one or more integrals that gives the total number of salamanders living in the cave.

Answer:
$$\int_{0}^{2/3} (x^2 - (6x^3 - 2x))\delta(x)dx$$