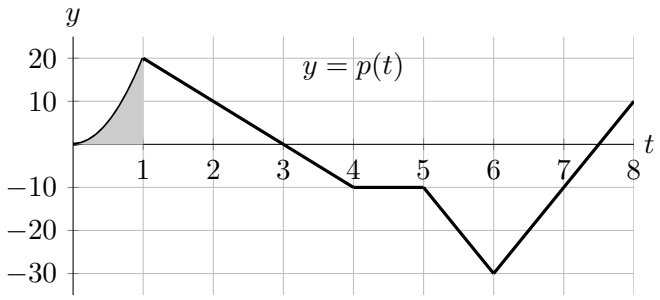
4. [8 points] In the cartoon adaptation of *Derivative Girl*, Darth Integrator dumps toxins (which will make everyone forget their calculus knowledge) into a local park's lake.

Derivative Girl discovers this October 10 at midnight (12am, 00:00), and tries her best to clear out the toxin in the lake. The rate at which the total volume of toxin in the lake changes, in gallons per day, is given by $p(t)$, where t is measured in days since October 10. The graph of $p(t)$ is shown below.



On October 15 at midnight, Derivative Girl realized there were still 50 gallons of toxin in the lake. Note that the shaded region in the graph above has area 7.

Let $P(t)$ be the total number of gallons of toxin in the lake t days after October 10 at midnight.

a. [2 points] How many days after Derivative Girl discovers the toxin is the volume of toxin in the lake at its largest?

Answer: 3

b. [2 points] How many days after Derivative Girl discovers the toxin is the amount of toxin in the lake growing fastest?

Answer: 1

c. [2 points] Write an expression involving one or more integrals that gives $P(t)$ in terms of the function p.

Answer: $P(t) = \int_5^t p(x)\,dx + 50$

d. [2 points] In the context of this problem, give a practical interpretation of the expression

$$\frac{1}{5-2}\int_2^5 P(t)\,dt.$$

Include units.

Solution: The average volume of toxin in the lake from October 12 to October 15 is $\frac{1}{5-2}\int_2^5 P(t)\,dt$ gallons.