

6. [7 points] Does the following integral converge or diverge? Be sure to show all work and indicate any theorems you use.

$$\int_{10}^{\infty} \frac{5x + \cos(x) - 1}{2x^3 + 2x + 7} dx$$

Answer (Circle one):

Diverges

Converges

**Justification:**

*Solution:* Since  $\cos(x) \leq 1$ , we have  $5x + \cos(x) - 1 \leq 5x$ .

We also know  $2x^3 + 2x + 7 \geq 2x^3$  for all  $x \geq 0$ . Therefore

$$\frac{5x + \cos(x) - 1}{2x^3 + 2x + 7} < \frac{5x}{2x^3} = \frac{5}{2x^2}$$

for  $x \geq 10$ .

By the  $p$ -test, with  $p = 2 > 1$ , we know that  $\int_{10}^{\infty} \frac{5}{2x^2} dx$  converges.

Therefore, by the (Direct) Comparison Test for improper integrals,  $\int_{10}^{\infty} \frac{5x + \cos(x) - 1}{2x^3 + 2x + 7} dx$  also converges.