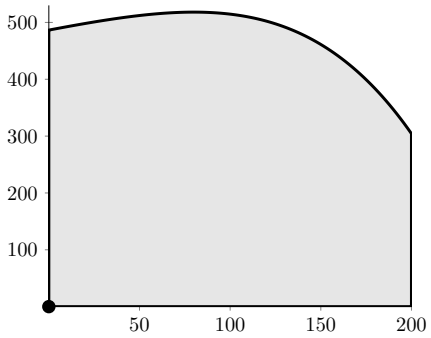
8. [11 points]

A city is designing a park, which will have three straight sides and one curved side, as shown in the diagram at right. There is a fountain at the southwest corner of the park, located at $(0, 0)$. Let $p(x)$ be the north-south distance, in feet, across the park x feet to the east of the fountain. Note that the park is 200 feet wide in the east-west direction.



City planners have the following data for $p(x)$.

x	0	50	100	150	200
$p(x)$	487	512	515	462	305
$p'(x)$	0.606	0.364	−0.373	−1.91	−4.57

a. [6 points] The planners would like to be able to say the following:

The area of the park is at most _____ square feet.

Given this, should they use an overestimate or an underestimate to approximate the area of the park?

Answer *(circle one)*: OVERESTIMATE UNDERESTIMATE

Which one of the following methods of approximation should they use to guarantee this?

RIGHT(n) LEFT(n) MID(n) TRAP(n)

Find the approximation you chose above, using the maximal amount of equal subintervals possible, for the area of the park in square feet. Write out all the terms in your sum.

Answer: $\text{MID}(2) = 100(p(50) + p(150)) = 100(512 + 462) = 97400$

b. [5 points] They plan to put a fence along the entire northern (curved) side. Write an expression involving one or more integrals that gives the total length of this fence in feet.

Answer: $\displaystyle\int_0^{200} \sqrt{1 + (p'(x))^2}\, dx$

Use a RIGHT(2) approximation to estimate the length of the fence. Write out all the terms in your sum.

Answer: $100(\sqrt{1 + (-0.373)^2} + \sqrt{1 + (-4.57)^2}) = 574.543$