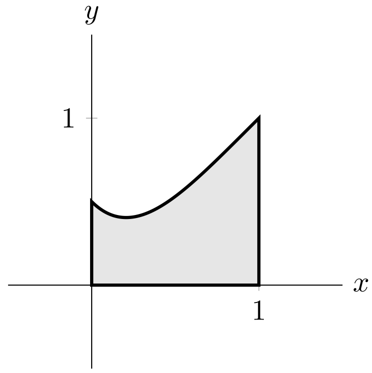
9. [12 points] Kyle wants to make a big ring, made by the rotation of the region bounded by

$$y = x + \frac{1}{2}(x-1)^4,\ x = 0,\ x = 1,\ \text{and}\ y = 0$$

about the line $x = -\frac{1}{2}$. This region is shown below. Both x and y are measured in centimeters.



a. [4 points] Write, but do not evaluate, an integral expression that gives the volume of Kyle's ring in cm^3.

Solution: It would be very difficult to slice with respect to y, since we would have to split the upper part into pieces. So we will slice with respect to x, which means we must use shell method.

Answer: $\int_0^1 2\pi\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}(x-1)^4\right)dx$

b. [4 points] The ring's density is given by $\ln(5r+1)$ grams/cm^3, where r is the distance in centimeters from the central axis of the ring. Write, but do not evaluate, an integral expressing the total mass of Kyle's ring in grams.

Solution: We must either write the density function in terms of the x-coordinate, or the integrand above in terms of r. Since the region is being rotated around $x = -\frac{1}{2}$, we get $r = x - -\frac{1}{2} = x + \frac{1}{2}$.

Answer: $\int_0^1 2\pi\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}(x-1)^4\right)\left(\ln\left(5(x+\frac{1}{2})+1\right)\right)dx$

c. [4 points] John wants to use the same region to make a ring, but instead rotates the region around the line $y = -\frac{1}{2}$.
Write, but do not evaluate, an integral that gives the **volume** of John's ring in cm^3.

Answer: $\int_0^1 \pi\left(\left(x+\frac{1}{2}(x-1)^4+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)dx$