1. [14 points] Let $f(x)$ be a differentiable function whose derivative $f'(x)$ is also differentiable. Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	0	1	2	3	4	5
$f(x)$	5	0	4	$\pi/2$	2	7
$f'(x)$	0	π	8	13	1	-6

Additionally, assume $f(x)$ is positive for $x > 2$.
Compute the exact value of the following integrals. If it is not possible to do so based on the information provided, then write 'NOT POSSIBLE' and clearly indicate why it is not possible. Show all of your work.

a. [4 points] $\displaystyle\int_1^2 \frac{f'(3x-1)}{f(3x-1)}\,dx$

Solution: Substitute $w = f(3x-1)$ to obtain

$$\int_{f(2)}^{f(5)} \frac{1}{3w}\,dw = \frac{1}{3}\ln|w|\Big|_4^7 = \frac{1}{3}\left(\ln(7)-\ln(4)\right) = \frac{1}{3}\ln\left(\frac{7}{4}\right)$$

b. [5 points] $\displaystyle\int_0^4 (x+1)f''(x)\,dx$

Solution: Integrating by parts,

$$\begin{aligned}\int_0^4 (x+1)f''(x)\,dx &= (x+1)f'(x)\Big|_0^4 - \int_0^4 f'(x)\,dx \\ &= \left((x+1)f'(x) - f(x)\right)\Big|_0^4 \\ &= 5f'(4) - f'(0) - f(4) + f(0) \\ &= 5 - 2 + 5 = 8\end{aligned}$$

c. [5 points] $\displaystyle\int_1^3 \left(\sin\left(f(x)\right)\right)^3 \cos\left(f(x)\right) f'(x)\,dx$

Solution: Either use the substitution $w = \sin(f(x))$ or $u = f(x)$ and then $w = \sin(u)$ to obtain

$$\begin{aligned}\int_{f(1)}^{f(3)} (\sin(u))^3\cos(u)\,du = \int_{\sin(f(1))}^{\sin(f(3))} w^3\,dw &= \frac{1}{4}w^4\Big|_{\sin(f(1))}^{\sin(f(3))} \\ &= \tfrac{1}{4}\left(\sin^4(\pi/2) - \sin^4(0)\right) \\ &= \tfrac{1}{4}\end{aligned}$$