5. [15 points] Consider the region $R$ in the $xy$-plane bounded between $y = \cos(x)$ and $y = -1$ for $x$ values between $-\pi$ and $\pi$. A sketch of the region is shown below.

a. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region $R$ around the line $x = 5$. Do not evaluate your integral(s).

*Solution:* Taking vertical slices, we see that we obtain the shell method. The volume of a slice of thickness $\Delta x$ at horizontal coordinate $x$ is approximately

$$2\pi(5 - x)(\cos(x) - (-1))\Delta x$$

and so the total volume of the solid is

$$2\pi \int_{-\pi}^{\pi} (5 - x)(\cos(x) + 1) \, dx.$$

b. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region $R$ around the line $y = -3$. Do not evaluate your integral(s).

*Solution:* Again using vertical slices, we obtain the washer method. The volume of a slice of thickness $\Delta x$ at horizontal coordinate $x$ is approximately

$$\pi \left((\cos(x) + 3)^2 - 2^2\right) \Delta x$$

and so the total volume of the solid is

$$\pi \int_{-\pi}^{\pi} ((\cos(x) + 3)^2 - 4) \, dx.$$
c. [5 points] Find an expression involving one or more integrals for the volume of the solid with a base in the shape of the region \( R \), and semicircular cross sections perpendicular to the \( x \)-axis. Do not evaluate your integral(s).

Solution: Again, take vertical slices. The area of a semi-circle is \( \frac{1}{2} \pi r^2 \) and the radius should be half the height of a slice. Putting this together, the volume of a slice of thickness \( \Delta x \) at horizontal coordinate \( x \) is approximately

\[
\frac{1}{2} \pi \left( \frac{1}{2} (\cos(x) + 1) \right)^2 \Delta x
\]

and so the total volume of the solid is

\[
\frac{1}{2} \pi \int_{-\pi}^{\pi} \left( \frac{1}{2} (\cos(x) + 1) \right)^2 \, dx
\]

\[
= \frac{\pi}{8} \int_{-\pi}^{\pi} (\cos(x) + 1)^2 \, dx
\]