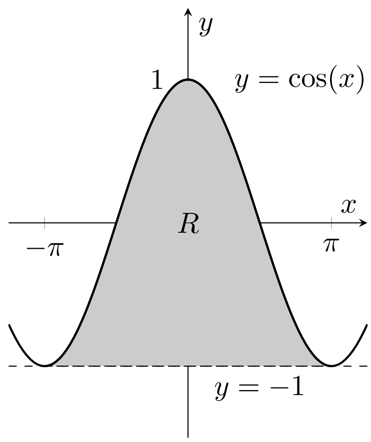
5. [15 points] Consider the region R in the xy−plane bounded between $y = \cos(x)$ and $y = -1$ for x values between $-\pi$ and π. A sketch of the region is shown below.



a. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $x = 5$. Do not evaluate your integral(s).

Solution: Taking vertical slices, we see that we obtain the shell method. The volume of a slice of thickness Δx at horizontal coordinate x is approximately

$$2\pi(5 - x)(\cos(x) - (-1))\Delta x$$

and so the total volume of the solid is

$$2\pi \int_{-\pi}^{\pi} (5 - x)(\cos(x) + 1)\ dx.$$

b. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $y = -3$. Do not evaluate your integral(s).

Solution: Again using vertical slices, we obtain the washer method. The volume of a slice of thickness Δx at horizontal coordinate x is approximately

$$\pi\left((\cos(x) + 3)^2 - 2^2\right)\Delta x$$

and so the total volume of the solid is

$$\pi \int_{-\pi}^{\pi} \left((\cos(x) + 3)^2 - 4\right)\ dx.$$

c. [5 points] Find an expression involving one or more integrals for the volume of the solid with a base in the shape of the region R, and semicircular cross sections perpendicular to the x-axis. Do not evaluate your integral(s).

Solution: Again, take vertical slices. The area of a semi-circle is $\frac{1}{2}\pi r^2$ and the radius should be half the height of a slice. Putting this together, the volume of a slice of thickness Δx at horizontal coordinate x is approximately

$$\frac{1}{2}\pi\left(\frac{1}{2}(\cos(x)+1)\right)^2\Delta x$$

and so the total volume of the solid is

$$\begin{aligned} & \frac{1}{2}\pi\int_{-\pi}^{\pi}\left(\frac{1}{2}(\cos(x)+1)\right)^2\,dx \\ = \; & \frac{\pi}{8}\int_{-\pi}^{\pi}(\cos(x)+1)^2\,dx \end{aligned}$$