6. [10 points] A thin circular plate of radius 3 m is being used to launch an electric rocket into space. The charge density, in Coulombs per m^2, on the surface of the plate a distance r meters from its center is given by a function $\delta(r) = 1 - kr$ for some constant k. Note that a Coulomb is a unit of electric charge.

a. [4 points] Write an expression involving integrals for the total charge, in Coulombs, on the surface of the circular plate. Do not evaluate the integral(s).

Solution: The charge on a thin circular ring of thickness Δr at a distance of r meters from the center is approximately

$$2\pi r(1 - kr)\Delta r$$

and so the total charge on the plate is given by

$$2\pi \int_0^3 r(1 - kr)\ dr.$$

b. [6 points] Find the value of k if the total charge on the surface of the plate is 3π Coulombs. Be sure to show all your work including algebra and any evaluation of integrals.

Solution: We have

$$\begin{aligned} 2\pi \int_0^3 r(1 - kr)\ dr &= 3\pi \\ 2\pi \left(\tfrac{1}{2}r^2 - \tfrac{1}{3}kr^3\right)\Big|_0^3 &= 3\pi \\ 2\pi \left(\tfrac{9}{2} - 9k\right) &= 3\pi \\ 9 - 18k &= 3 \\ k &= \tfrac{1}{3}. \end{aligned}$$