7. [15 points] Consider the function

$$f(x) = \frac{1}{(x-2)^2(x-3)}$$

a. [4 points] Approximate the integral $\int_{-5}^{1} f(x)\,dx$ using MID(3). Do not decompose $f(x)$ before doing the approximation. Write out each term in any sums you make.

Solution:

$$\begin{aligned} \text{MID}(3) &= 2\left(f(-4)+f(-2)+f(0)\right) \\ &= 2\left(-\tfrac{1}{7(6^2)} - \tfrac{1}{5(4^2)} - \tfrac{1}{3(2^2)}\right) \end{aligned}$$

b. [4 points] Approximate the integral $\int_{-5}^{1} f(x)\,dx$ using TRAP(3). Do not decompose $f(x)$ before doing the approximation. Write out each term in any sums you make.

Solution:

$$\begin{aligned} \text{TRAP}(3) = {} & \frac{1}{2}(2f(-5)+2f(-3)) + \frac{1}{2}(2f(-3)+2f(-1)) + \frac{1}{2}(2f(-1)+2f(1)) \\ = {} & -\frac{1}{8(7^2)} - \frac{2}{6(5^2)} - \frac{2}{4(3^2)} - \frac{1}{2(1^2)} \end{aligned}$$

Alternatively, you may take the average of LEFT(3) and RIGHT(3) to obtain the same result.

c. [7 points] Split the function $f(x)$ into partial fractions with two or more terms. Do not integrate these terms.

Solution: Let

$$\frac{1}{(x-2)^2(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)}.$$

Then

$$1 = A(x-2)(x-3) + B(x-3) + C(x-2)^2.$$

Comparing coefficients of x^2,

$$A + C = 0, \text{ so } C = -A.$$

Comparing coefficients of x,

$$-5A + B - 4C = 0, \text{ so } B = A.$$

Comparing coefficients of 1,

$$6A - 3B + 4C = 1, \text{ so } A = -1.$$

Substituting back, we see that $B = -1$ and $C = 1$, so

$$\frac{1}{(x-2)^2(x-3)} = \frac{-1}{(x-2)} + \frac{-1}{(x-2)^2} + \frac{1}{(x-3)}.$$