8. [8 points]

a. [4 points] Kesha is tasked with sourcing mahogany timber for her logging company. She finds that the cost of producing $x$ tons of timber is given by $M(x)$, measured in thousands of dollars. She knows that $M'(x) = \ln(1 + x^2)$ and that it costs 11 thousand dollars to produce 3 tons of timber. Find an expression involving an integral for $M(x)$.

Solution:

$$M(x) = 11 + \int_3^x \ln(1 + t^2) \, dt$$

b. [4 points] Kesha finds that her department’s profit for the month, in thousands of dollars, is given by

$$\int_{-5}^{5} \sin(x^5) + 2 \, dx.$$ 

By evaluating this integral, give a numerical figure for Kesha’s department’s profit. (Hint: What special property of the function $\sin(x^5)$ could be useful here?) Make sure to fully justify your answer.

Solution:

$$\int_{-5}^{5} \sin(x^5) + 2 \, dx = \int_{-5}^{5} \sin(x^5) \, dx + 2 \int_{-5}^{5} 1 \, dx = \int_{-5}^{5} \sin(x^5) \, dx + 20$$

But $\sin(x^5)$ is an odd function since $\sin((-x)^5) = -\sin(x^5)$ (both $x^5$ and $\sin(x)$ are odd). This means that $\int_{-5}^{5} \sin(x^5) \, dx = 0$, and so the profit is 20 thousand dollars.