8. [8 points]

a. [4 points] Kesha is tasked with sourcing mahogany timber for her logging company. She finds that the cost of producing x tons of timber is given by $M(x)$, measured in thousands of dollars. She knows that $M'(x) = \ln(1 + x^2)$ and that it costs 11 thousand dollars to produce 3 tons of timber. Find an expression involving an integral for $M(x)$.

Solution:

$$M(x) = 11 + \int_3^x \ln(1 + t^2)\ dt$$

b. [4 points] Kesha finds that her department's profit for the month, in thousands of dollars, is given by

$$\int_{-5}^{5} \sin(x^5) + 2\ dx.$$

By evaluating this integral, give a numerical figure for Kesha's department's profit. (Hint: What special property of the function $\sin(x^5)$ could be useful here?) Make sure to fully justify your answer.

Solution:

$$\begin{aligned}\int_{-5}^{5} \sin(x^5) + 2\ dx &= \int_{-5}^{5} \sin(x^5)\ dx + \int_{-5}^{5} 2\ dx \\ &= \int_{-5}^{5} \sin(x^5)\ dx + 20\end{aligned}$$

But $\sin(x^5)$ is an odd function since $\sin((-x)^5) = \sin(-(x^5)) = -\sin(x^5)$ (both x^5 and $\sin(x)$ are odd). This means that $\int_{-5}^{5} \sin(x^5)\ dx = 0$, and so the profit is 20 thousand dollars.