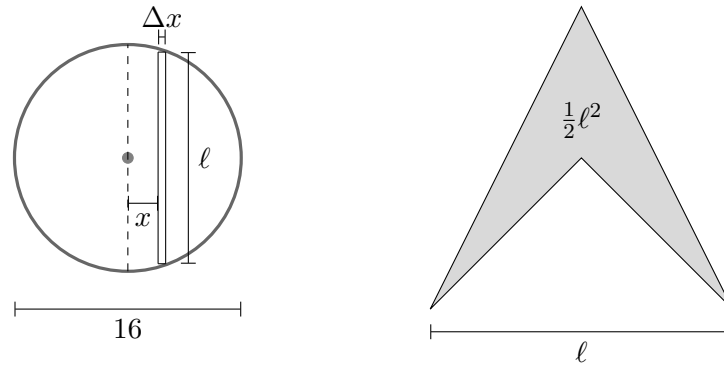


5. [11 points] Tony's climbing gym wants to put in a climbing structure based off of the Chicago Bean. However, they want to make it more angular. The base of the structure will be in the shape of a circle with an 8 meter radius. The cross-sections perpendicular to the circle lying above a slice of the circle of length  $\ell$  meters (as shown below) have area  $\frac{1}{2}\ell^2$  square meters and are pictured below. The density of the material used to build the structure is not constant and has density dependent on its horizontal distance  $x$  from the vertical diameter through the circle. The density in  $\text{kg}/\text{m}^3$  is given by  $\delta(x) = 1000\sqrt{1+x^2}$ .



- a. [2 points] Write an expression that gives the quantity  $\ell$  in terms of  $x$ .
- b. [3 points] Write an expression that gives the approximate volume, in cubic meters, of a slice of the structure a horizontal distance  $x$  meters away from the diameter of the circle with thickness  $\Delta x$ . Your expression should not involve any integrals.
- c. [3 points] Using your expression from (b) to write an expression involving integrals which gives the total volume of the structure in cubic meters. Do not evaluate any integrals.
- d. [3 points] Write an expression involving integrals which gives the total mass of the structure in kg. Your answer may contain  $\delta(x)$ . Do not evaluate any integrals.