1. [15 points] Below are a table of values for a function $f(x)$ which is **odd** and twice differentiable.

x	0	1	2	3	4
$f(x)$	0	2	-1	4	1
$f'(x)$	1	5	e	2	0

Use the table to compute the following quantities. Show your work.

a. [4 points] Approximate the integral $\displaystyle\int_{-1}^{1} f(2x+2)dx$ using MID(2). Write out each term in your sum.

Solution: Set $g(x) = f(2x+2)$. Then,

$$\text{MID}(2) = 1\big(g(-\tfrac{1}{2}) + g(\tfrac{1}{2})\big) = f(1) + f(3) = 2 + 4 = 6.$$

b. [4 points] $\displaystyle\int_{-3}^{3} f'(x)(2x+2)dx$.

Solution: Using integration by parts($u = 2x+2$ and $dv = f'(x)dx$) and the fact that f is odd:

$$\begin{aligned}\int_{-3}^{3} f'(x)(2x+2)dx &= (2x+2)f(x)\Big|_{-3}^{3} - 2\int_{-3}^{3} f(x)dx \\ &= 8f(3) - (-4)(f(-3)) \\ &= 8(4) - (-4)(-4) = 16.\end{aligned}$$

Note that since f is odd, $\displaystyle\int_{-3}^{3} f(x)dx = 0$.

c. [3 points] $\displaystyle\int_{-1}^{1} (x+1)f'((x+1)^2)dx$.

Solution: Substitute $w = (x+1)^2$ to obtain

$$\int_{-3}^{3} (x+1)f'((x+1)^2)dx = \frac{1}{2}\int_{0}^{4} f'(w)dw = \frac{1}{2}f(w)\Big|_{0}^{4} = \frac{1}{2}(f(4) - f(0)) = \frac{1}{2}(1-0) = \frac{1}{2}.$$

d. [4 points] The average value of $(f(x)+1)^2 f'(x)$ on $[2,4]$.

Solution: Substitute $w = f(x)$ to obtain

$$\frac{1}{2}\int_{2}^{4} (f(x)+1)^2 f'(x)dx = \frac{1}{2}\int_{f(2)}^{f(4)} (w+1)^2 dw = \frac{(w+1)^3}{6}\Big|_{-1}^{1} = \frac{1}{6}(8-0) = \frac{4}{3}.$$