3. [13 points] Miley and Kylie see their friend Brian in the distance and decide to race to see who can reach him first. However, they see Brian begin pacing back and forth so depending on when they start the race, they will run a different amount. The distance they run, in meters, if the race starts t seconds after Brian begins pacing is

$$L(t) = 25 + 4 \int_{-\left(\frac{\pi}{2}\right)^{\frac{1}{3}}}^{t^3} \cos(r^3) r^2 dr$$

Throughout this problem, please give answers in **exact form** and include **units.**

a. [4 points] If Miley and Kylie start the race immediately as Brian begins pacing, what distance will they run? Evaluate any integrals in your answer and remember to include units.

Solution: The initial value is $L(0)$. Substituting $w = r^3$,

$$\begin{aligned} L(0) &= 25 + 4 \int_{-\left(\frac{\pi}{2}\right)^{\frac{1}{3}}}^{0} \cos(r^3) r^2 dr \\ &= 25 + \frac{4}{3} \int_{-\frac{\pi}{2}}^{0} \cos w dw = 25 + \frac{4}{3}\left(\sin w \Big|_{-\frac{\pi}{2}}^{0} \right) \\ &= 25 + \frac{4}{3}(0 - (-1)) = 25 + \frac{4}{3} \text{ meters.} \end{aligned}$$

b. [6 points] Miley and Kylie decide they will start the race at the smallest strictly positive time t (i.e. smallest t with $t > 0$) such that $L'(t) = 0$. Find the time at which they will start the race. Make sure to include units.

Solution: Taking the derivative using the chain rule,

$$L'(t) = 4\cos((t^3)^3)(t^3)^2 3t^2 = 12t^8 \cos(t^9).$$

Since $t^8 \geq 0$ for $t \geq 0$, the smallest strictly positive time for which $L'(t) = 0$ occurs at the smallest positive time for which the cosine term is 0. This happens when $t^9 = \frac{\pi}{2}$. Solving this gives that the race will start at $t = \left(\frac{\pi}{2}\right)^{\frac{1}{9}}$ seconds.

c. [3 points] Miley and Kylie want to be able to compute $L(t)$ quickly, so they would like $L(t)$ rewritten in the form below. Write $L(t)$ in the form given below with appropriate expressions in place of the blanks.

$$L(t) = \underline{\quad 25 + \tfrac{4}{3} \quad} + \int_0^t \underline{\quad 12r^8 \cos(r^9) \quad}\, dr$$