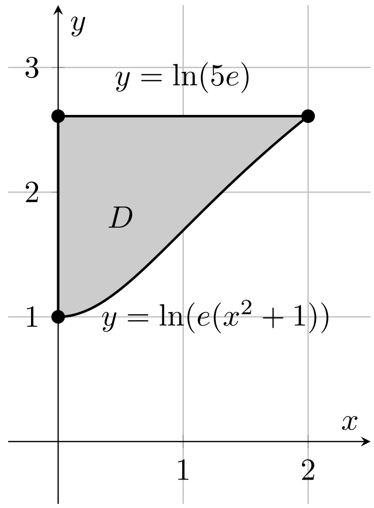
4. [16 points] Consider the region D in the $xy-$plane bounded between $y = \ln(e(x^2+1))$ and $y = \ln(5e)$ for x values between 0 and 2. A sketch of the region is shown below.



a. [6 points] Using the washer method, find an expression involving one or more integrals for the volume of the solid formed by rotating the region D around the x-axis. Do not evaluate your integral(s).

Solution: Taking vertical slices, we see that we obtain the washer method. The volume of a slice of thickness Δx at horizontal coordinate x is approximately

$$\Delta V = \pi((\ln(5e))^2 - (ln(e(x^2+1)))^2)\Delta x$$

and so the total volume of the solid is

$$\int_0^2 \pi((\ln(5e))^2 - (ln(e(x^2+1)))^2)dx.$$

b. [6 points] Using the shell method, find an expression involving one or more integrals for the volume of the solid formed by rotating the region D around the line $x = 2$. Do not evaluate your integral(s).

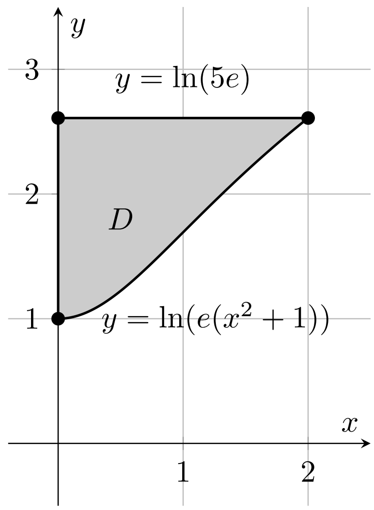
Solution: Taking vertical slices, we see that we obtain the shell method. Since we are rotating around $x = 2$, the horizontal distance between x and the axis of rotation is $2-x$ for any $x \in [0,2]$. The volume of a slice of thickness Δx at horizontal coordinate x is approximately

$$\Delta V = 2\pi(2-x)(\ln(5e) - \ln(e(x^2+1)))\Delta x$$

and so the total volume of the solid is

$$\int_0^2 2\pi(2-x)(\ln(5e) - \ln(e(x^2+1)))dx.$$

4. (continued) Here is a reproduction of the plot on the previous page:



c. [4 points] Find an expression involving one or more integrals for the perimeter of the region D. Do not evaluate your integral(s).

Solution: The boundary of the region D has 3 sides. The left side of D has length $\ln(5e) - 1 = \ln(5)$. The top side of D has length 2. To calculate the length of the lower curve, set $f(x) = \ln(e(x^2 + 1))$. Then

$$f'(x) = \frac{2x}{x^2 + 1}.$$

Using the arc length formula, we get that the perimeter is

$$\ln(5) + 2 + \int_0^2 \sqrt{1 + \left(\frac{2x}{x^2 + 1}\right)^2}\, dx.$$