

6. [13 points]

- a. [6 points] Split the function $\frac{4-9x}{(x-2)^2(x+5)}$ into partial fractions with 2 or more terms. **Do not integrate these terms.** Please show all work to obtain your partial fractions.

Solution: Let

$$\frac{4-9x}{(x-2)^2(x+5)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+5}.$$

Then,

$$A(x-2)(x+5) + B(x+5) + C(x-2)^2 = 4-9x.$$

Comparing coefficients of x^2

$$A + C = 0, \text{ so } A = -C.$$

Comparing coefficients of x ,

$$3A + B - 4C = -9, \text{ so } B = -9 - 7A.$$

Comparing the coefficients of the constant term,

$$4C + 5B - 10A = 4, \text{ so } A = -1.$$

Substituting back, we see that $B = -2$ and $C = 1$, so

$$\frac{4-9x}{(x-2)^2(x+5)} = \frac{-1}{x-2} + \frac{-2}{(x-2)^2} + \frac{1}{x+5}.$$

- b. [7 points] Use the fact that $\frac{5x}{(x^2+1)(x-2)} = \frac{2}{x-2} + \frac{-2x+1}{x^2+1}$ to solve the indefinite integral

$$\int \frac{5x}{(x^2+1)(x-2)} dx.$$

Solution: We first split the integral into 3 terms and use substitution $w = x^2 + 1$ on the second term:

$$\begin{aligned} \frac{5x}{(x^2+1)(x-2)} dx &= \int \frac{2}{x-2} dx + \int \frac{-2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \\ &= \int \frac{2}{x-2} dx - \int \frac{1}{w} dw + \int \frac{1}{x^2+1} dx \\ &= 2 \ln|x-2| - \ln|w| + \arctan x + C \\ &= 2 \ln|x-2| - \ln(x^2+1) + \arctan x + C. \end{aligned}$$

Note that we have substituted back in for x .