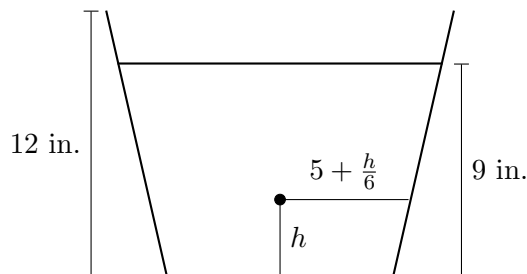


8. [10 points] Frank, an aspiring chef and looking to impress his extended family, makes a big pot of tomato soup that he brings to his family reunion potluck. The pot is 12 inches tall with **circular cross sections** parallel to the bottom of the pot. The cross section h inches from the bottom of the pot has radius $5 + \frac{h}{6}$ inches for $0 \leq h \leq 12$. Unfortunately, his least favorite cousin Barrie brought a better tomato soup to the potluck. Almost no one ate Frank's soup and after the potluck, the pot still had soup up to 9 inches above the bottom of the pot. Frank saved the rest of the soup for himself and now he has to get the remaining soup out of the pot.



- a. [3 points] Write an expression for the volume of a thin horizontal slice of soup at height h from the bottom of the pot and thickness Δh . Make sure to include units.

Solution: Taking our horizontal slice the volume ΔV can be expressed using the fact that the radius $r = 5 + \frac{h}{6}$:

$$\Delta V = \pi r^2 \Delta h = \pi \left(5 + \frac{h}{6}\right)^2 \Delta h \text{ in.}^3$$

- b. [4 points] By the end of the potluck, the soup has settled into an uneven density. The density of the soup at height h above the bottom of the pot is $.05(1+h)$ pounds/in³. Write an expression for the amount of work in pound-inches required to get a thin horizontal slice of soup at height h above the bottom of the pot and thickness Δh to the top of the pot.

Solution: The thin horizontal slice at height h above the bottom of the pot and thickness Δh must be raised $12 - h$ inches. It has density $.05(1+h)$ pounds/in³. So, the work ΔW done in raising the horizontal slice is

$$\Delta W = .05(1+h)\pi\left(5 + \frac{h}{6}\right)^2(12-h)\Delta h.$$

- c. [3 points] Write a definite integral that represents the total amount of work in pound-inches required to get all the soup that was left in the pot after the potluck out of the pot. Do not evaluate the integral.

Solution: The pot is 9 in. full, so the total work done in raising the soup is

$$\int_0^9 .05\pi(1+h)\left(5 + \frac{h}{6}\right)^2(12-h)dh.$$