1. [16 points] Use the table to compute the following quantities. The function $h(x)$ is **odd**, **twice differentiable**, and $h'(x) > 0$ for all x-values. Write your answer using exact form on the blank provided. If there is not enough information available to answer the question, write N.I. You need to evaluate all integrals, but you do not need to simplify your final answer.

x	0	1	2	3	4
$h(x)$	0	2	4	5	7
$h'(x)$	7	3	4	7	2

a. [4 points] $\displaystyle\int_3^4 \frac{h''(t)}{h'(t)}\,dt$

Answer: $\ln(2/7)$.

Solution: Let $u = h'(t)$, so $du = h''(t)dt$ and the integral becomes $\displaystyle\int_7^2 u^{-1}\,du = \ln(2) - \ln(7)$ which is $\ln(2/7)$.

b. [4 points] The average value of $h'(x)$ on $[-1, 1]$

Answer: **2**.

Solution: The average value of $h'(x)$ on $[-1, 1]$ is by definition $\displaystyle\frac{1}{1-(-1)}\int_{-1}^{1} h'(x)\,dx$. Using the fundamental theorem of calculus, this is equal to $\frac{1}{2}(h(1) - h(-1))$. Since $h(x)$ is odd, $h(-1) = -h(1) = -2$, so the answer is $\frac{1}{2}(2 - (-2)) = 2$.

c. [4 points] $\displaystyle\int_1^4 (w+1)h''(w)\,dw$

Answer: -1.

Solution: Since integration splits over addition, this is equal to $\displaystyle\int_1^4 wh''(w)\,dw + \int_1^4 h''(w)\,dw$. For the first integral we integrate by parts to obtain $\displaystyle\int_1^4 wh''(w)\,dw = -\int_1^4 h'(w)\,dw + wh'(w)|_1^4 = h(1) - h(4) + 4h'(4) - h'(1) = 0$. For the second integral we use FTC to get $\displaystyle\int_1^4 h''(w)\,dw = h'(4) - h'(1) = 2 - 3 = -1$.

d. [4 points] $\displaystyle\int_{1/2}^{2} x^{-1/2} h'(\sqrt{2x})\,dx$

Answer: $2\sqrt{2}$.

Solution: Let $u = \sqrt{2x} = (2x)^{1/2}$. By the chain rule and power rule, $du = \frac{1}{\sqrt{2x}}dx$. Therefore $dx = \sqrt{2x}du$. So the integral given is equal to $\sqrt{2}\int_1^2 h'(u)\,du = \sqrt{2}(h(2) - h(1)) = 2\sqrt{2}$.