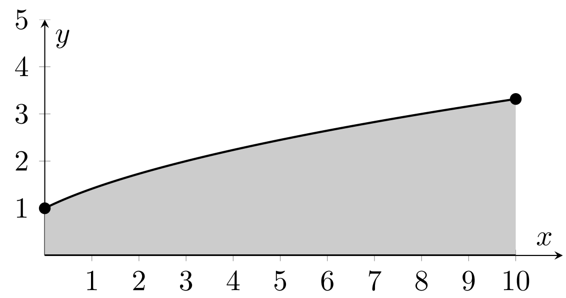
6. [10 points] Denise and Trystan are undersea research scientists, and they are preparing to descend into the ocean in a newly-constructed submarine. The submarine's shape is given by rotating the region below the curve $y = \sqrt{x+1}$, above the x-axis, and between $x = 0$ and $x = 10$ (see figure) about the x-axis. Here, x and y are measured in meters.

Graph of $y = \sqrt{x+1}$ from $x = 0$ to $x = 10$



The density of the submarine is not constant, due to the advanced materials used in its construction. Instead, the density $p(x)$ varies, and is given by $p(x) = (x-5)^2 + 1$ kg/m^3.

a. [5 points] Write an expression for the **volume** of a slice of the submarine at position x and of thickness Δx. Include units.

Solution: The radius of such a slice is given by $r(x) = \sqrt{x+1}$, so the volume is $\pi(r(x))^2\Delta x = \pi(x+1)\Delta x$ m^3.

b. [2 points] Write an expression for the **mass** of the slice you found in part (a). Include units.

Solution: The density function $p(x)$ depends only on x, so the density is roughly constant on the slice from part (a), as long as Δx is very small. The mass of such a slice is then

$$M(x) = p(x) \cdot \pi(x+1)\Delta x = [(x-5)^2 + 1]\pi(x+1)\Delta x \text{ kg.}$$

c. [3 points] Write, but do not evaluate, an integral which gives the **total mass** of the submarine. Include units.

Solution: The approximate mass of the submarine is obtained by adding together all the masses of the slices calculated above to get $\sum[(x-5)^2+1]\pi(x+1)\Delta x$. In the limit we get the exact mass in the form of an integral:

$$\int_0^{10}[(x-5)^2+1]\pi(x+1)\,dx.$$