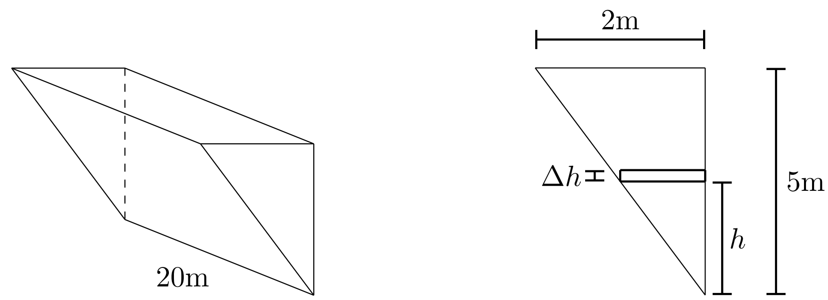
7. [13 points] A drinking water facility needs to pump water out of an underground tank. The tank is 20 meters in length with right-triangular cross-sections perpendicular to the ground as shown in the figure. The top of the tank is a 2m by 20m rectangle. The **top** of the tank lies **5 meters below the surface of the earth.** Recall that $g = 9.8\text{m/s}^2$, where g is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the **volume** (in cubic meters) of a horizontal rectangular slice of water at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

Solution: The length of the slice is 20m. Call the width w. To find w in terms of h, we use similar triangles (using the diagram on the right) to set up the proportion:

$$\frac{w}{h} = \frac{2}{5} \quad \Longrightarrow \quad w = \frac{2}{5}h.$$

Therefore the volume of such a slice is (we use that the volume of a rectangular prism is its length times its width times its height):

$$20 \cdot \frac{2}{5}h \cdot \Delta h = 8h\Delta h.$$

b. [2 points] The density of water is approximately 1000 kg/m^3. Write an expression for the **weight** (in Newtons) of the slice of water from part (a). Your answer should not involve an integral.

Solution: The density is constant, and therefore the mass of such a slice is the volume of that slice times 1000 kg/m^3, which is $8000h\Delta h$. To obtain the weight in Newtons, we multiply by $g = 9.8\text{m/s}^2$ to get $9.8 \cdot 8000h\Delta h$.

c. [3 points] Write an expression for the **work** (in Joules) needed to pump the slice of water (from parts (a) and (b)) to the surface of the earth. Your answer should not involve an integral.

Solution: (Note: the wording of this problem was slightly edited for clarity) The slice lies $5 - h$ meters below the top of the tank, and the top of the tank is 5 meters below the surface of the earth, so the total distance we move the slice up is $10 - h$ meters. Therefore the work (in Joules) done to move one slice up to the surface of the earth is approximately

$$(10 - h) \cdot (9.8 \cdot 8000h\Delta h).$$

d. [3 points] Assuming the tank is initially full of water, write an integral for the **total work** (in Joules) needed to pump all of the water to the surface of the earth.

Solution: Adding up the contributions of the work needed to move each slice found in part (c) and taking a limit as the thickness Δh of each slice goes to zero, we obtain the exact answer in the form of the integral

$$\int_0^5 (10 - h)(9.8 \cdot 8000h)\, dh.$$