9. [10 points]

a. [6 points] As a particle moves in the xy plane, it traces out the curve $y = g(x)$ for $1 \le x \le 5$, where $g(x)$ is the function

$$g(x) = \int_{2x}^{x^4} \sin(t^3)\, dt + 100e^{\pi}.$$

Set up, but do not evaluate, an expression with a *single* integral which gives the arclength of the path of the particle. Your answer should not involve the letter g.

Solution: The arclength formula says that the length of the path of the particle is given by

$$\int_1^5 \sqrt{1 + (g'(x))^2}\, dx.$$

The FTC gives

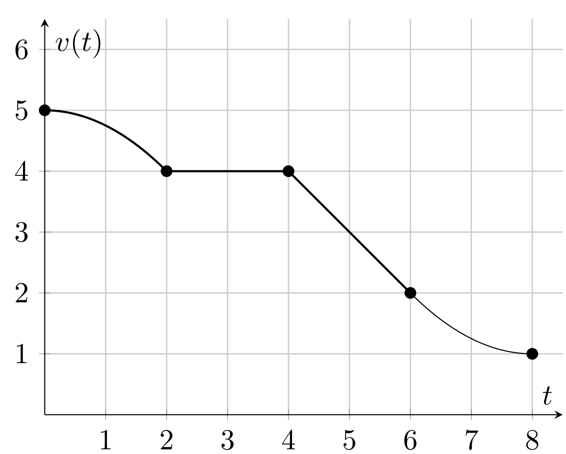
$$g'(x) = 4x^3 \sin(x^{12}) - 2\sin(8x^3).$$

Therefore, the answer is

$$\int_1^5 \sqrt{1 + \left[4x^3 \sin(x^{12}) - 2\sin(8x^3)\right]^2}\, dx.$$

b. [4 points] A different particle is traveling with velocity given by $v(t)$ meters/second, where $v(t)$ is the function in the following graph: The units of t are seconds.

Graph of $y = v(t)$



Using a left Riemann sum with four subdivisions, estimate the distance the particle travels in the first 8 seconds of its journey. Is this an underestimate or an overestimate of the actual distance traveled in the first 8 seconds? Justify your answer.

Solution: Since the length of each subdivision is 2, we obtain:

$$\begin{aligned}\text{LEFT}(4) &= 2 \cdot 5 + 2 \cdot 4 + 2 \cdot 4 + 2 \cdot 2 \\ &= 2(5 + 4 + 4 + 2) \\ &= 30.\end{aligned}$$

This is an overestimate of the actual distance. This is because the function $v(t)$ is decreasing on $[0, 2]$ and $[4, 8]$ and constant on $[2, 4]$. Left Riemann sums overestimate the actual area for decreasing functions.