1. [15 points] Let $f(x)$ be a differentiable function whose derivative $f'(x)$ is also differentiable and is always positive. Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	1	2	3	4	5	6
$f(x)$	2	7	8	12	15	17
$f'(x)$	3	2	6	11	4	5

Additionally, you are given that $\displaystyle\int_2^4 \frac{f(x)}{x}\,dx = 9$.

Compute the exact value of the following integrals. If there is not enough information provided to determine the value of the integral, write "NEI" and clearly indicate why. Show all of your work.

a. [5 points] $\displaystyle\int_1^2 \big(f'(t)+4\big)\,e^{f(t)+4t}\,dt$

Solution: Substitute $w = f(t)+4t$ to obtain

$$\int_{f(1)+4}^{f(2)+8} e^w\,dw = e^w\Big|_6^{15} = e^{15}-e^6.$$

Answer: $e^{15}-e^6$

b. [5 points] $\displaystyle\int_2^4 f'(x)\ln x\,dx$

Solution: Integrate by parts to obtain

$$\int_2^4 f'(x)\ln x\,dx = f(x)\ln x\Big|_2^4 - \int_2^4 \frac{f(x)}{x}\,dx = 12\ln 4 - 7\ln 2 - 9 = 17\ln 2 - 9,$$

where we have used the fact that $\displaystyle\int_2^4 \frac{f(x)}{x}\,dx = 9$.

Answer: $12\ln 4 - 7\ln 2 - 9$

c. [5 points] $\displaystyle\int_{\ln 2}^{\ln 4} f(e^x)\,dx$

Solution: Substitute $w = e^x$ to obtain

$$\int_2^4 \frac{f(w)}{w}\,dw = 9,$$

where we have again used the fact that $\displaystyle\int_2^4 \frac{f(x)}{x}\,dx = 9$.

Answer: 9