3. [9 points] Anna and Burt have come to an agreement after Labor Day's food debacle. They've decided to cook lasagna for their family's next get-together. They practice cooking the lasagna over the course of 4 hours. Let $L(t)$ be the tastiness of the lasagna, measured in tasty units, t hours after they begin cooking. $L(t)$ is given by

$$L(t) = \int_1^{t^2-3t+3} \frac{7}{1+x^4}\,\mathrm{d}x + 3, \text{ for } 0 \le t \le 4.$$

a. [2 points] There are exactly two times within the interval $[0, 4]$ where the lasagna is 3 tasty units. What are those times? Show your work.

Solution: $L(t) = 3$ when the upper bound and lower bound are equal. Solving the equation $t^2 - 3t + 3 = 1$ yields $t = 1, 2$.

Answer: $t = 1, 2$

b. [4 points] During what interval(s) in $[0, 4]$ is the lasagna's tastiness decreasing? Justify your answer(s) using calculus.

Solution: Solving $L'(t) = \dfrac{7}{1+(t^2-3t+3)^4}(2t-3) = 0$ gives the unique critical point $t = 3/2$. If $t < 3/2$, then $L'(t) < 0$ (the opposite is true if $t > 3/2$). Therefore, tastiness is decreasing on the interval $[0, 3/2)$.

Answer: $[0, 3/2)$

c. [3 points] Find a function $f(x)$ and constants a and C so that we may rewrite $L(t)$ in the form

$$L(t) = \int_a^t f(x)\,\mathrm{d}x + C.$$

There may be more than one correct answer.

$f(x) = \dfrac{7(2x-3)}{1+(x^2-3x+3)^4}$ $\qquad a = 1$ $\qquad C = 3$