4. [9 points] As Megan's assortment of mushrooms continues to grow, she starts tracking the growth of various mushrooms. She finds that one mushroom has an erratic growth rate. Its growth rate t days after it blooms is given by the function

$$m(t) = \frac{10\cos(t)}{(\sin^2(t)+1)\,(\sin(t)+2)} + 6 \quad \text{for } 0 \le t \le 5,$$

measured in centimeters per day.
The height of Megan's mushroom 5 days after it blooms is given by the integral

$$\int_0^5 m(t)\,\mathrm{d}t.$$

Evaluate this integral, showing all your work. Give an exact answer and include units. You may use the fact that

$$\frac{1}{(u^2+1)(u+2)} = \frac{2-u}{5\,(u^2+1)} + \frac{1}{5\,(u+2)}.$$

Solution: Via substitution method, with $u = \sin(t)$,

$$\int_0^5 \frac{10\cos(t)}{(\sin^2(t)+1)\,(\sin(t)+2)}\,\mathrm{d}t = \int_0^{\sin(5)} \frac{10}{(u^2+1)(u+2)}\,\mathrm{d}u.$$

Using the fact given in the problem, we have

$$\int_0^5 m(t)\,\mathrm{d}t = 4\int_0^{\sin(5)} \frac{1}{u^2+1}\,\mathrm{d}u - 2\int_0^{\sin(5)} \frac{u}{u^2+1}\,\mathrm{d}u + 2\int_0^{\sin(5)} \frac{1}{u+2}\,\mathrm{d}u + 30.$$

Using $v = u^2+1$, we have

$$\int_0^{\sin(5)} \frac{u}{u^2+1}\,\mathrm{d}u = \frac{1}{2}\int_1^{\sin^2(5)+1} \frac{1}{v}\,\mathrm{d}v.$$

Therefore,

$$\int_0^5 m(t)\,\mathrm{d}t = 4\arctan(\sin(5)) - \ln(\sin^2(5)+1) + 2(\ln(\sin(5)+2) - \ln(2)) + 30.$$

The units are centimeters.

Answer: $4\arctan(\sin(5)) - \ln(\sin^2(5)+1) + 2\ln(\sin(5)+2) - 2\ln(2) + 30$