5. [9 points] Jinho is playing a game involving a deck of cards, when he notices that the backs of the cards are painted in an unusual way. Jinho finds that the total mass of paint on the back of a card, in grams, can be expressed as

$$\int_3^6 (10 - g(12 - 2x))\ dx$$

where the function $g(x)$ is decreasing and concave up. Some of the values of $g(x)$ are given in the following table:

x	0	1	2	3	4	5	6
$g(x)$	6	4	2.5	1.5	1.0	0.75	0.7

a. [3 points] Using RIGHT(2), find an approximation for the total mass of paint, measured in grams, on the back of the card. Write out all the terms in your sum. You do not need to simplify.

Solution: The interval $[3, 6]$ has width 3, so we should divide it into two subintervals of width 1.5. This means we should plug $x = 4.5$ and $x = 6$ into the function $10 - g(12 - 2x)$. We obtain:

$$\begin{aligned} \text{RIGHT}(2) &= 1.5\,(10 - g(12 - 2(4.5)) + 10 - g(12 - 2(6))) \\ &= 1.5(20 - g(12 - 9) - g(12 - 12)) \\ &= 1.5(20 - g(3) - g(0)) \\ &= 1.5(20 - 1.5 - 6) \\ &= 1.5(12.5) = 18.75. \end{aligned}$$

b. [3 points] Using MID(3), find an approximation for the total mass of paint, measured in grams, on the back of the card. Write out all the terms in your sum. You do not need to simplify.

Solution: The interval $[3, 6]$ has width 3, so we should divide it into three subintervals of width 1. This means we should plug $x = 3.5$, $x = 4.5$ and $x = 5.5$ into the function $10 - g(12 - 2x)$. We obtain:

$$\begin{aligned} \text{MID}(3) &= 10 - g(12 - 2(3.5)) + 10 - g(12 - 2(4.5)) + 10 - g(12 - 2(5.5)) \\ &= 30 - g(12 - 7) - g(12 - 9) - g(12 - 11) \\ &= 30 - g(5) - g(3) - g(1) \\ &= 30 - 0.75 - 1.5 - 4 \\ &= 23.75. \end{aligned}$$

c. [3 points] Is the MID(3) estimate to the total mass of paint you found in part (b) an underestimate, an overestimate, or is there not enough information? Circle your choice and briefly explain your answer.

Circle one: UNDERESTIMATE **OVERESTIMATE** NOT ENOUGH INFORMATION

Explanation:

Solution: We must determine whether the function $f(x) := 10 - g(12 - 2x)$ is concave up or concave down. We see, using the chain rule, that $f'(x) := 2g'(12 - 2x)$ and $f''(x) := -4g''(12 - 2x)$. Since $g(x)$ is concave up, $f(x)$ must be concave down, and so MID must always give an overestimate.