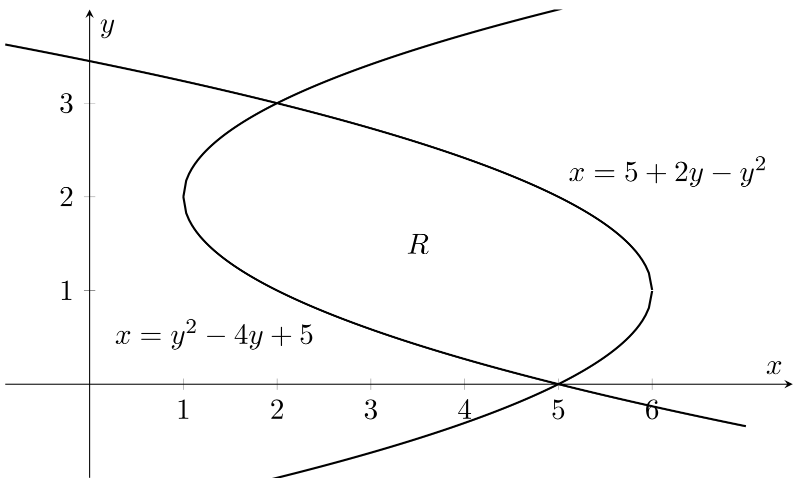
6. [15 points] The curves $x = y^2 - 4y + 5$ and $x = 5 + 2y - y^2$ intersect at the points $(2, 3)$ and $(5, 0)$, as seen in the diagram below. Consider the region, R, bounded by the two curves.



a. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $x = 0$ (i.e. the y-axis). Do not evaluate your integral(s).

Solution: We use horizontal slices, which gives rise to washers. For this region, y ranges between 0 and 3, so we get

$$\int_0^3 \pi \left((5+2y-y^2)^2 - (y^2-4y+5)^2\right)\, dy = \int_0^3 4\pi y(y-5)(y-3)\, dy$$

Answer: ______________________________

b. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $y = 4$. Do not evaluate your integral(s).

Solution: We use horizontal slices, which gives rise to cylindrical shells. For this region, y ranges between 0 and 3, so we get

$$\begin{aligned}\int_0^3 2\pi\,(4-y)\left((5+2y-y^2) - (y^2-4y+5)\right)\, dy &= \int_0^3 2\pi\,(4-y)\left(6y-2y^2\right)\, dy \\ &= \int_0^3 4\pi y\,(4-y)\,(3-y)\, dy\end{aligned}$$

Answer: ______________________________

c. [5 points] Find an expression involving one or more integrals for the volume of the solid which has the region R as its base, and which has square cross-sections perpendicular to the y-axis. Do not evaluate your integral(s).

Solution: We use horizontal slices. If a cross-sectional slice has width s, then its area is s^2. For this region, y ranges between 0 and 3, so we get

$$\int_0^3 \left((5+2y-y^2)-(y^2-4y+5)\right)^2 \, dy = \int_0^3 4y^2\,(y-3)^2 \, dy$$

Answer: ______________________________