6. [15 points] The curves \( x = y^2 - 4y + 5 \) and \( x = 5 + 2y - y^2 \) intersect at the points \((2, 3)\) and \((5, 0)\), as seen in the diagram below. Consider the region, \( R \), bounded by the two curves.

![Diagram of the curves and region R](image)

a. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region \( R \) around the line \( x = 0 \) (i.e. the \( y \)-axis). Do not evaluate your integral(s).

**Solution:** We use horizontal slices, which gives rise to washers. For this region, \( y \) ranges between 0 and 3, so we get

\[
\int_0^3 \pi \left( (5 + 2y - y^2)^2 - (y^2 - 4y + 5)^2 \right) \, dy = \int_0^3 4\pi y(y - 5)(y - 3) \, dy
\]

**Answer:**

b. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region \( R \) around the line \( y = 4 \). Do not evaluate your integral(s).

**Solution:** We use horizontal slices, which gives rise to cylindrical shells. For this region, \( y \) ranges between 0 and 3, so we get

\[
\int_0^3 2\pi (4 - y) \left( (5 + 2y - y^2) - (y^2 - 4y + 5) \right) \, dy = \int_0^3 2\pi (4 - y) \left( 6y - 2y^2 \right) \, dy
\]

\[
= \int_0^3 4\pi y (4 - y) (3 - y) \, dy
\]

**Answer:**

c. [5 points] Find an expression involving one or more integrals for the volume of the solid which has the region \( R \) as its base, and which has square cross-sections perpendicular to the \( y \)-axis. Do not evaluate your integral(s).
Solution: We use horizontal slices. If a cross-sectional slice has width \( s \), then its area is \( s^2 \). For this region, \( y \) ranges between 0 and 3, so we get

\[
\int_0^3 \left( (5 + 2y - y^2) - (y^2 - 4y + 5) \right)^2 \, dy = \int_0^3 4y^2 (y - 3)^2 \, dy
\]

Answer: ________________________________