7. [7 points] Not content with rolling a whole boulder up a hill for all of eternity, Sisyphus instead opts to break up his punishment boulder into smaller pieces of rock and lift them up the hill inside a bucket.

Suppose Sisyphus builds a platform at the top of the hill that is 15 feet above the ground. He lifts the bucket vertically from ground level to the platform. Unfortunately, the bucket has a hole where rocks can fall out.

a. [3 points] Let $W(y)$ be the weight of the bucket with rocks, in pounds, when it is y feet **above the ground**. Write an expression involving one or more integrals for the total work done to lift the bucket up to the platform. Your answer should involve $W(y)$. Do not evaluate your integral(s). Include units.

Answer: $\displaystyle\int_0^{15} W(y)\,dy$ **Units:** ftlbs

b. [4 points] Sisyphus lifts the bucket up at a constant rate of 2 feet per second. The weight of the bucket with rocks decreases at a rate of

$$r(t) = \frac{10}{1+e^{-t}}$$

pounds per second, where t is measured in seconds since Sisyphus started lifting the bucket. Assume the bucket and the rocks together weigh 100 pounds initially. Find a formula for $W(y)$ involving one or more integrals. Do not evaluate your integral(s).

Solution: As the bucket is lifted at a constant rate, the height of the bucket after t seconds is $y = 2t$, and so $t = \frac{y}{2}$. The total decrease in weight in the first $\frac{y}{2}$ seconds is given by integrating the rate of change from 0 to $\frac{y}{2}$. Therefore, remembering the initial weight is 100 pounds, we see that the weight after lifting the bucket y feet is given by

$$W(y) = 100 - \int_0^{y/2} \frac{10}{1+e^{-t}}\,dt$$

Answer: $\displaystyle W(y) = 100 - \int_0^{y/2} \frac{10}{1+e^{-t}}\,dt$