9. [7 points] Let $g(x)$ be a function that is twice-differentiable for all x. Additionally, $g(x)$ has the following properties:

- $g(x)$ has no inflection points on the interval $(0, 10)$
- $g'(x)$ does not change signs on the interval $(0, 10)$
- $g'(5) = 1$
- $g''(7) = -2$

Define the function $G(x)$ to be

$$G(x) = \int_1^x g(t)\,dt.$$

a. [2 points] Is $G(x)$ concave up, concave down, or neither at $x = 9$? No justification is required.

Circle one: **[CONCAVE UP]** CONCAVE DOWN NEITHER

Solution: (Not required). $G'(x) = g(x)$ by construction theorem. So $G''(x) = g'(x)$. $g'(9) > 0$ because $g'(5) = 1$ and the $g'(x)$ has no sign changes on the interval $(0, 10)$. Therefore, $G''(9) > 0$ and $G'(x)$ is concave up at $x = 9$.

b. [5 points] With the blanks provided, order from least-to-greatest

$$G(9), \quad \text{LEFT}(9), \quad \text{RIGHT}(9), \quad \text{MID}(9), \quad \text{TRAP}(9)$$

where all the approximations above are of the definite integral $G(9)$. No justification is required.

__________ $\leq$ __________ $\leq$ __________ $\leq$ __________ $\leq$ __________

Solution: Since $g(t)$ is increasing and concave down, we have:

$$\text{LEFT}(9) \leq \text{TRAP}(9) \leq G(9) \leq \text{MID}(9) \leq \text{RIGHT}(9)$$